$\boxed{P1}$ $r(t) = \left(R e^{-at} \operatorname{Cos}(t), R e^{-at} \operatorname{Sin}(t)\right), \quad t \in [0, 4\pi]$

$a > 0, \; R > 0$

<u>Suave</u>: exp, Cos, Sin son C^1 y Algebra de C^1 ✓

<u>Regular</u>: $\frac{dr}{dt} = \left(R e^{-at}(-a \operatorname{Cos}(t) - \operatorname{Sin}(t)), R e^{-at}(-a \operatorname{Sin}(t) + \operatorname{Cos}(t))\right)$

$$\left\|\frac{dr}{dt}\right\| = \sqrt{R^2 e^{-2at}\left(a^2 \operatorname{Cos}^2(t) + 2a \operatorname{Cos}(t)\operatorname{Sin}(t) + \operatorname{Sin}^2(t)\right) + R^2 e^{-2at}\left(a^2 \operatorname{Sin}^2(t) - 2a \operatorname{Sin}(t)\operatorname{Cos}(t) + \operatorname{Cos}^2(t)\right)}$$

$$= R e^{-at}\sqrt{a^2(\operatorname{Cos}^2(t) + \operatorname{Sin}^2(t)) + (\operatorname{Cos}^2(t) + \operatorname{Sin}^2(t))}$$

$$= R e^{-at}\sqrt{a^2+1} > 0 \quad ✓$$

<u>Simple</u>: Sean t_1, t_2 tales que:

$$r(t_1) = r(t_2) \Rightarrow R e^{-at_1} \operatorname{Cos}(t_1) = R e^{-at_2} \operatorname{Cos}(t_2) \quad (1)$$

$$R e^{-at_1} \operatorname{Sin}(t_1) = R e^{-at_2} \operatorname{Sin}(t_2) \quad (2)$$

$$(1)^2 + (2)^2 \Rightarrow \cancel{R^2} e^{-2at_1} = \cancel{R^2} e^{-2at_2} \quad / \ln(\;)$$

$$\Rightarrow -2at_1 = -2at_2 \Rightarrow t_1 = t_2 \rightarrow \text{Simple} ✓$$

<u>Longitud</u>
<u>$t \in [0, 4\pi]$</u>

$$\int_0^t R e^{-at}\sqrt{a^2+1}\, dt = \left.\frac{R e^{-at}}{-a}\sqrt{a^2+1}\right|_0^t$$

$$\Rightarrow S(t) = \frac{R\sqrt{a^2+1}}{a}(1 - e^{-at})$$

P2 Clases hartos ejemplos

P3 a) $M = \int_0^{2\pi} \rho(r(t)) \cdot \left\|\frac{dr}{dt}\right\| dt$, $\rho(r(t)) = a^2 \cos^2(t)$

$\frac{dr}{dt} = \left(-a\sin(t), a\cos(t), \frac{h}{2\pi}\right)$

$\Rightarrow \left\|\frac{dr}{dt}\right\| = \sqrt{a^2 + \frac{h^2}{4\pi^2}}$

$$\Rightarrow M = \int_0^{2\pi} a^2 \cos^2(t) \sqrt{a^2 + \frac{h^2}{4\pi^2}}\, dt = a^2\sqrt{a^2+\frac{h^2}{4\pi^2}} \int_0^{2\pi} \frac{1+\cancelto{0}{\cos(2t)}}{2} dt$$

$$= \boxed{a^2 \sqrt{a^2 + \frac{h^2}{4\pi^2}} \cdot \pi}$$

$$X_G \cdot M = \int_0^{2\pi} x(t) \cdot \rho(r(t)) \cdot \left\|\frac{dr}{dt}\right\| dt$$

$$= \int_0^{2\pi} a^3 \cos^3(t) \sqrt{a^2 + \frac{h^2}{4\pi^2}}\, dt = a^3 \sqrt{a^2+\frac{h^2}{4\pi^2}} \int_0^{2\pi} \cancelto{0}{\cos(t)} - \sin^2(t)\cos(t)\, dt$$

$$= a^3 \sqrt{a^2 + \frac{h^2}{4\pi^2}} \left(-\frac{\sin^3(t)}{3}\right)\Bigg|_0^{2\pi} = \boxed{0} \Rightarrow \boxed{X_G = 0}$$

$$Y_G \cdot M = \int_0^{2\pi} y(t) \rho(r(t)) \left\|\frac{dr}{dt}\right\| dt = \int_0^{2\pi} a^3 \cos^2(t) \sin(t) \sqrt{a^2 + \frac{h^2}{4\pi^2}}\, dt$$

$$= a^3 \sqrt{a^2 + \frac{h^2}{4\pi^2}} \left(-\frac{\cos^3(t)}{3}\right)\Bigg|_0^{2\pi} = 0 \Rightarrow \boxed{Y_G = 0}$$

$$M \cdot Z_G = \int_0^{2\pi} z(t) \rho(r(t)) \left\|\frac{dr}{dt}\right\| dt = \int_0^{2\pi} a^2 \cos^2(t) \cdot t \cdot \frac{h}{2\pi} \sqrt{a^2 + \frac{h^2}{4\pi^2}}\, dt = \frac{a^2 h}{2\pi}\sqrt{a^2+\frac{h^2}{4\pi^2}} \overbrace{\int_0^{2\pi} t \cdot \cos^2(t)\, dt}^{I}$$

$$I = \underbrace{\int_0^{2\pi} \frac{t}{2}}_{=\pi^2} + \frac{t\cos(2t)}{2}\, dt = \pi^2 + \frac{1}{2}\int_0^{2\pi} t\cos(2t) = \pi^2 + \frac{1}{2}\left(\cancel{t\frac{\sin(2t)}{2}}\Bigg|_0^{2\pi} - \int_0^{2\pi} \cancelto{0}{\frac{\sin(2t)}{2}}\, dt\right)$$

$$M z_G = \frac{a^2 h}{2} \pi \sqrt{a^2 + \frac{h^2}{4\pi^2}}$$

$$\Rightarrow \boxed{z_G = \frac{h}{2}} \Longrightarrow \boxed{\text{Centro } G = \left(0, 0, \frac{h}{2}\right)}$$

b) $r(t) = (r\cos(t), r\sin(t)) \Rightarrow \left\| \frac{dr}{dt} \right\| = r$, $t \in [0, 2\pi]$

$$\rho(x,y) = \begin{cases} 2m & y > 0 \\ m & y \le 0 \end{cases}$$

$$\Rightarrow \rho(r(t)) = \begin{cases} 2m & t \in (0, \pi) \\ m & t \in [\pi, 2\pi] \end{cases}$$

$$M = \int_0^{2\pi} \rho(r(t)) \left\| \frac{dr}{dt} \right\| dt = \int_0^{\pi} \rho(r(t)) \cdot r\, dt + \int_{\pi}^{2\pi} \rho(r(t)) \cdot r\, dt$$

$$= \int_0^{\pi} 2mr\, dt + \int_{\pi}^{2\pi} m \cdot r\, dt = \boxed{3\pi m r}$$

$$X_G \cdot M = \int_0^{2\pi} r\cos(t) \cdot \rho(r(t))\, r\, dt = \int_0^{\pi} r\cos(t) \cdot 2m \cdot r\, dt + \int_{\pi}^{2\pi} r\cos(t)\, m\, r\, dt$$

$$= 2mr^2 \cdot \underbrace{\cancel{\sin(t)}\Big|_0^{\pi}}_{0} + mr^2 \underbrace{\cancel{\sin(t)}\Big|_{\pi}^{2\pi}}_{0} = 0 \Rightarrow \boxed{X_G = 0}$$

$$Y_G \cdot M = \int_0^{2\pi} r\sin(t)\, \rho(r(t)) \cdot r\, dt = \int_0^{\pi} r\sin(t) \cdot 2m \cdot r\, dt + \int_{\pi}^{2\pi} r\sin(t) \cdot m \cdot r\, dt$$

$$= -2mr^2\cos(t)\Big|_0^{\pi} - mr^2\cos(t)\Big|_{\pi}^{2\pi} = 4mr^2 - 2mr^2 = 2mr^2$$

$$\Rightarrow \boxed{Y_G = \frac{2}{3} r} \Rightarrow \text{Centro } \left(0, \frac{2}{3} r\right)$$

Propuesta: Cambio $\rho(x,y) = m$

$$\Rightarrow \rho(r(t)) = m$$

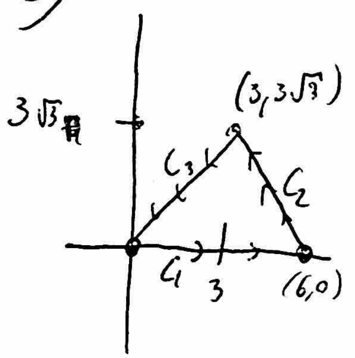
$$M = \int_0^{2\pi} m \cdot r \, dt = 2\pi r m$$

$$X_G \cdot M = \int_0^{2\pi} \cancel{r}\cos(t) \cdot m \cdot r \, dt = \cancel{r}^2 \cdot m \cdot \sin(t)\Big|_0^{2\pi} = 0 \Rightarrow \boxed{X_G = 0}$$

$$Y_G \cdot M = \int_0^{2\pi} r \sin(t) \cdot m \cdot r \, dt = r^2 m(-\cos(t))\Big|_0^{2\pi} = 0 \Rightarrow \boxed{Y_G = 0}$$

es el origen

c)



Error enunciado con $(0,0)$, $(6,0)$, $(3,3\sqrt{2})$

$\rho(r(t)) = m$ (Uniforme)

$C_1 \Rightarrow r_{C_1} = (t,0)$, $t \in [0,6]$

$C_2 \Rightarrow r_{C_2} = t(3,3\sqrt{3}) + (1-t)\cdot(6,0)$, $t \in [0,1]$

$$= (6-3t,\ 3\sqrt{3}\cdot t)$$

$C_3 \Rightarrow r_{C_3} = t\cdot(0,0) + (1-t)(3,3\sqrt{3})$ $\quad t \in [0,1]$

$$= (3(1-t),\ 3\sqrt{3}(1-t))$$

$$M = \int_{C_1} \rho(r(t)) \cdot \left\|\frac{dr}{dt}\right\| dt + \int_{C_2} \rho(r(t)) \left\|\frac{dr}{dt}\right\| + \int_{C_3} \rho(r(t)) \left\|\frac{dr}{dt}\right\| dt$$

$$= \int_0^6 m \cdot \overbrace{\|(1,0)\|}^{1} dt + \int_0^1 m \cdot \overbrace{\|(-3,3\sqrt{3})\|}^{6} dt + \int_0^1 m \cdot \overbrace{\|(-3,-3\sqrt{3})\|}^{6} dt$$

$$= 6 \cdot m \cdot 1 + m \cdot 6 \cdot 1 + m \cdot 6 \cdot 1 = \boxed{18m}$$

$$M\cdot X_G = \int_{C_1} t\cdot m\cdot 1\,dt + \int_{C_2} (6-3t)\cdot m\cdot 6\,dt + \int_{C_3} 3(1-t)\cdot m\cdot 6\,dt$$

$$= \frac{m}{2}t^2\Big|_0^6 + 6m\left(6t-\frac{3t^2}{2}\right)\Big|_0^1 + 18m\left(t-\frac{t^2}{2}\right)\Big|_0^1$$

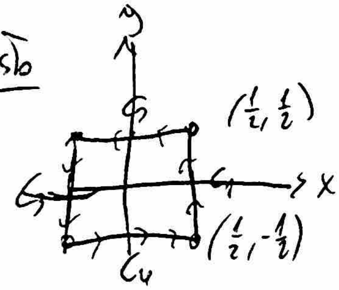
$$= 18m + 27m + 9m = 54\cdot m \Rightarrow \boxed{X_G = 3}$$

$$M\cdot Y_G = \int_0^6 0\cdot m\cdot 1\,dt + \int_0^1 3\sqrt{3}\cdot t\cdot m\cdot 6\,dt + \int_0^1 3\sqrt{3}\,(1-t)\cdot m\cdot 6\,dt$$

$$= 0 + 18\sqrt{3}\,m\,\frac{t^2}{2}\Big|_0^1 + 18\sqrt{3}\cdot m\left(t-\frac{t^2}{2}\right)\Big|_0^1$$

$$= 9\sqrt{3}\,m + 9\sqrt{3}\cdot m = 18\sqrt{3}\cdot m \Rightarrow \boxed{Y_G = \sqrt{3}}$$

<u>Propuesto</u>



$\Gamma_{C_1} = (\frac{1}{2}, t)$, $t\in[-\frac{1}{2},\frac{1}{2}] \Rightarrow \frac{d\Gamma}{dt} = (0,1)$

$\Gamma_{C_2} = (\frac{1}{2}-t, \frac{1}{2})$, $t\in[0,1]$ 〃 $= (-1,0)$

$\Gamma_{C_3} = (-\frac{1}{2}, \frac{1}{2}-t)$, $t\in[0,1]$ 〃 $= (0,-1)$

$\Gamma_{C_4} = (t-\frac{1}{2}, -\frac{1}{2})$, $t\in[0,1]$ 〃 $= (1,0)$

Todas Norma 1

$\rho(\Gamma(t)) = m$

$$M = \int_{-\frac{1}{2}}^{\frac{1}{2}} m\cdot 1\,dt + \int_0^1 m\cdot 1\,dt + \int_0^1 m\cdot 1\,dt + \int_0^1 m\cdot 1\,dt = \boxed{4m}$$

$$X_G\cdot M = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{2}\cdot m\cdot 1\,dt + \int_0^1 \left(\frac{1}{2}-t\right)\cdot m\,dt + \int_0^1 \left(-\frac{1}{2}\right)\cdot m\,dt + \int \left(t-\frac{1}{2}\right)m\,dt = 0$$

$\Rightarrow X_G = 0$, son justo las contrarias

$$Y_G\cdot M = \int_{-\frac{1}{2}}^{\frac{1}{2}} t\cdot m\,dt + \int_0^1 \frac{1}{2}\,m\,dt + \int_0^1 \left(\frac{1}{2}-t\right)\cdot m\,dt + \int_0^1 \left(-\frac{1}{2}\right)m\,dt = \frac{t^2}{2}\cdot m\Big|_{-\frac{1}{2}}^{\frac{1}{2}} + \frac{m}{2} - \frac{t^2}{2}m\Big|_0^1$$

$$= 0 \Rightarrow \boxed{Y_G = 0}$$