$\boxed{P1}$ $r(\gamma,\mu) = (\sin(\gamma), \mu, \cos(\gamma))$

$$\left.\begin{aligned} \frac{\partial r}{\partial \gamma} &= (\cos(\gamma), 0, -\sin(\gamma)) \\ \frac{\partial r}{\partial \mu} &= (0, 1, 0) \end{aligned}\right\}$$

$$\Rightarrow \frac{\partial r}{\partial \gamma} \times \frac{\partial r}{\partial \mu} = \left| \begin{array}{ccc:cc} \hat{i} & \hat{j} & \hat{k} & \tilde{i} & \hat{j} \\ \cos(\gamma) & 0 & -\sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \right|$$

$$= (\sin(\gamma), 0, \cos(\gamma))$$

$\left\| \frac{\partial r}{\partial \gamma} \times \frac{\partial r}{\partial \mu} \right\| = 1 \Rightarrow$ Es regular

$\Rightarrow \hat{n} = (\sin(\gamma), 0, \cos(\gamma))$

$\boxed{P2}$ a)

$$\left.\begin{aligned} &\varphi(r,\theta) = (r\cos(\theta), r\sin(\theta), r^2\cos(\theta)\sin(\theta)) \\ &\underbrace{r \in [0,\sqrt{2}],\ \theta \in [0, 2\pi]}_{D} \end{aligned}\right\} \varphi(D) = S \checkmark$$

$$\Rightarrow \frac{\partial \varphi}{\partial r} = (\cos\theta, \sin\theta, 2r\cos(\theta)\sin\theta)$$

$$\frac{\partial \varphi}{\partial \theta} = (-r\sin\theta, r\cos\theta, r^2(\cos^2\theta - \sin^2\theta))$$

$$\frac{\partial \varphi}{\partial r} \times \frac{\partial \varphi}{\partial \theta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos\theta & \sin\theta & 2r\cos\theta\sin\theta \\ -r\sin\theta & r\cos\theta & r^2(\cos^2\theta - \sin^2\theta) \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{matrix}$$

$$= \Big(r^2(\cos^2\theta - \sin^2\theta)\sin\theta - 2r^2\cos^2\theta\sin\theta \, ,$$

$$-2r^2\cos\theta\sin^2\theta - r^2(\cos^2\theta - \sin^2\theta)\cos\theta \, ,$$

$$r\cos^2\theta + r\sin^2\theta \Big)$$

$$= \left(-r^2\sin\theta(\cos^2\theta + \sin^2\theta), -r^2\cos\theta(\cos^2\theta + \sin^2\theta), r \right)$$

$$= \left(-r^2\sin\theta \, , -r^2\cos\theta \, , r \right)$$

$$\left\| \frac{\partial \varphi}{\partial r} \times \frac{\partial \varphi}{\partial \theta} \right\| = \left(\sqrt{r^2+1} \right) r$$

$$\Rightarrow \text{Area}(S) = \iint_D \left\| \frac{\partial \varphi}{\partial r} \times \frac{\partial \varphi}{\partial \theta} \right\| \, dr \, d\theta$$

$$= \int_0^{\sqrt{2}} \int_0^{2\pi} r\left(\sqrt{r^2+1}\right) d\theta \, dr = \frac{2\pi}{3} \left(r^2+1\right)^{\frac{3}{2}} \Big|_0^{\sqrt{2}}$$

$$= \boxed{\frac{2\pi}{3}\left(3^{\frac{3}{2}} - 1 \right)}$$

b) Hay 2 opciones (Por lo menos)

Opción 1 $\quad \psi(u,v) = (u-v, u+v, uv)$, $\quad u \in [-1,1]$

$v \in [-\sqrt{1-u^2}, \sqrt{1-u^2}]$

Opción 2 $\quad \psi(r,\theta) = (r(\cos\theta - \sin\theta), r(\cos\theta + \sin\theta), r^2 \cos\theta \sin\theta)$

$r \in [0,1]$

$\theta \in [0, 2\pi]$

Encuentro más cómodo 2 para circulos

$$\frac{\partial \psi}{\partial r} = (\cos\theta - \sin\theta, \cos\theta + \sin\theta, 2r\cos\theta\sin\theta)$$

$$\frac{\partial \psi}{\partial \theta} = (-r(\cos\theta + \sin\theta), r(\cos\theta - \sin\theta), r^2(\cos^2\theta - \sin^2\theta))$$

$$\frac{\partial \psi}{\partial r} \times \frac{\partial \psi}{\partial \theta} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ \cos\theta - \sin\theta & \cos\theta + \sin\theta & 2r\cos\theta\sin\theta & \cos\theta - \sin\theta & \cos\theta + \sin\theta \\ -r(\cos\theta + \sin\theta) & r(\cos\theta - \sin\theta) & r^2(\cos^2\theta - \sin^2\theta) & -r(\cos\theta + \sin\theta) & r(\cos\theta - \sin\theta) \end{vmatrix}$$

$$= \Big(r^2(\cos^3\theta - \cos\theta\sin^2\theta + \cos^2\theta\sin\theta - \sin^3\theta) - 2r^2(\cos^2\theta\sin\theta + \sin^2\theta\cos\theta),$$
$$-2r^2(\cos^2\theta\sin\theta + \sin^2\theta\cos\theta) - r^2\cos^3\theta + r^2\sin^2\theta\cos\theta + r^2\cos^2\theta\sin\theta - r^2\sin^3\theta,$$
$$r(\cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta) + r(\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta) \Big)$$

$$= \left(r^2(\cos^3\theta - \sin^3\theta + \cos\theta\sin^2\theta + \cos^2\theta\sin\theta),\ -r^2(\cos^3\theta + \sin^3\theta + \cos\theta\sin^2\theta + \sin\theta\cos^2\theta),\ 2r \right)$$

$$= \left(r^2(\cos\theta - \sin\theta),\ -r^2(\cos\theta + \sin\theta),\ 2r \right)$$

$$\left\| \frac{\partial \varphi}{\partial r} \times \frac{\partial \varphi}{\partial \theta} \right\| = \sqrt{2r^4 + 4r^2} = r\sqrt{2r^2 + 4}$$

$$\Rightarrow \text{Area} = \int_0^1 \int_0^{2\pi} r\sqrt{2r^2+4}\, d\theta\, dr$$

$$= 2\pi \int_0^1 \frac{4r}{4} \sqrt{2r^2+4}\, dr \qquad \Big/ \begin{array}{l} u = 2r^2+4 \\ du = 4r\,dr \end{array}$$

$$= 2\pi \int_4^6 \frac{\sqrt{u}}{4}\, du$$

$$= \frac{\pi}{2} \frac{2}{3} u^{\frac{3}{2}} \Big|_4^6 = \boxed{\frac{\pi}{3}\left(6^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)}$$

$\boxed{P.3}$ $\varphi(r,\theta) = (r\cos\theta,\ r\sin\theta,\ 4 + r\cos\theta + r\sin\theta)$

$r \in [0,2]$

$\theta \in [0, 2\pi]$

$\varphi(D) = S$

$$\frac{\partial \varphi}{\partial r} = (\cos\theta,\ \sin\theta,\ \cos\theta + \sin\theta)$$

$$\frac{\partial \varphi}{\partial \theta} = (-r\sin\theta,\ r\cos\theta,\ -r\sin\theta + r\cos\theta)$$

$$\left(\frac{\partial\varphi}{\partial r}\times\frac{\partial\varphi}{\partial\theta}\right) = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ \cos\theta & \sin\theta & \cos\theta+\sin\theta & \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta & r(\cos\theta-\sin\theta) & -r\sin\theta & r\cos\theta \end{array}\right|$$

$$= \Big(r(\cos\theta\sin\theta - \sin^2\theta) - r(\cos^2\theta + \cos\theta\sin\theta),$$

$$-r(\cos\theta\sin\theta + \sin^2\theta) - r(\cos^2\theta + \sin\theta\cos\theta),$$

$$r(\cos^2\theta + \sin^2\theta)\Big)$$

$$= (-r, -r, r)$$

$$\left\|\frac{\partial\varphi}{\partial r}\times\frac{\partial\varphi}{\partial\theta}\right\| = \sqrt{3}\,r$$

$$\Rightarrow \iint_S (x^2z+y^2z)\,dS = \int_0^2\int_0^{2\pi} r^2(4 + r\cos\theta + r\sin\theta)\sqrt{3}\,r\,d\theta\,dr$$

$$= \int_0^2 r^3\sqrt{3}\left(\int_0^{2\pi} 4 + r\cancelto{0}{\cos\theta} + r\cancelto{0}{\sin\theta}\,d\theta\right)dr$$

$$= 2\pi r^4\sqrt{3}\Big|_0^2 = \boxed{32\sqrt{3}\,\pi}$$