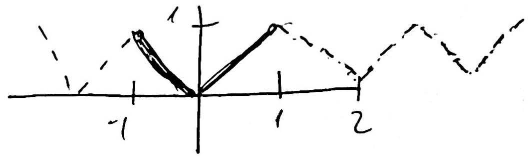
Pauta Auxiliar 10

P1

a) $f(x)=|x|$



$$a_0=\frac{1}{1}\int_{-1}^{1} f(x)\,dx = 1\cdot\int_{-1}^{1}|x|\,dx \overset{Par}{=} 2\cdot\int_{0}^{1}|x|\,dx = 2\int_{0}^{1}x\,dx = x^2\Big|_0^1 = \boxed{1}$$

$$\Rightarrow \boxed{\frac{a_0}{2}=\frac{1}{2}}$$

$$a_N=\frac{1}{1}\int_{-1}^{1} f(x)\cdot \operatorname{Cos}\left(N\cdot\frac{\pi}{1}x\right)dx = \int_{-1}^{1}|x|\cdot\operatorname{Cos}(N\pi x)\,dx \overset{Par}{=} 2\int_{0}^{1}x\cdot\operatorname{Cos}(N\pi x)\,dx$$

$$= 2\cdot\left[\underbrace{\left(x\cdot\frac{\operatorname{Sin}(N\pi x)}{N\pi}\right)\Big|_0^1}_{0} - \int_0^1 \frac{\operatorname{Sin}(N\pi x)}{N\pi}\,dx\right] = 2\cdot\frac{\operatorname{Cos}(N\pi x)}{(N\pi)^2}\Big|_0^1$$

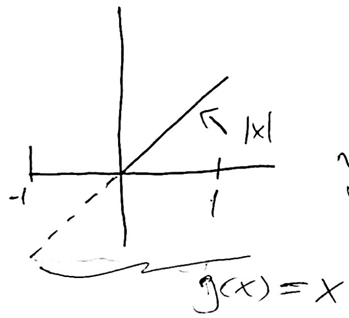
$$= \frac{2}{(N\pi)^2}\left[\operatorname{Cos}(N\pi)-1\right] = \frac{2}{(N\pi)^2}\left((-1)^N-1\right) = \begin{cases} 0 & \text{Si } N \text{ es par} \\ \dfrac{-4}{(N\pi)^2} & \text{Si } N \text{ es impar} \end{cases}$$

$$b_N=\frac{1}{1}\int_{-1}^{1} f(x)\cdot\operatorname{Sin}(N\pi x)\,dx = \int_{-1}^{1}|x|\operatorname{Sin}(N\pi x)\,dx \overset{Impar}{=} 0$$

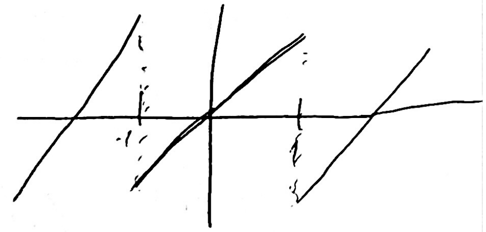
$$\Rightarrow f(x) = \frac{1}{2} + \sum_{N=1}^{\infty} a_N\cdot\operatorname{Cos}(N\pi x) = \frac{1}{2} + \sum_{K=1}^{\infty}\frac{-4}{(2K-1)^2\pi^2}\cdot\operatorname{Cos}((2K-1)\pi x)$$

Ver enlace de Ucursos

b) Estan pidiendo la extensión impar, es decir,



y por lo tanto la serie sera



$$a_0 = \int_{-1}^{1} g(x)\, dx = 0$$

$a_N = 0$ pues es impar

$$b_N = 2\int_0^1 x \cdot \sin(N\pi x)\, dx = 2\left[x\cdot\left(-\frac{\cos(N\pi x)}{N\pi}\right)\Bigg|_0^1 + \int_{-1}^{1} \frac{\cos(N\pi x)}{N\pi}\, dx \right]$$

es par

$$= -2\,\frac{(-1)^N}{N\pi}$$

$$\Rightarrow g(x) \approx \sum_{N=1}^{\infty} \frac{2(-1)^{N+1}}{N\pi} \cdot \sin(N\pi x)$$

Se Aproxima $g(x)$ salvo en los extremos, pues

$$Sg(-1) = Sg(1) = \frac{g(-1) + g(1)}{2} = 0$$

, tambien ver enlaces

c) Notar que es lo mismo que la parte a)

d) Se sabe que en a) la $S_f(x)$ Aproxima bien $f(x)$ por lo que evaluando en $\boxed{x=0}$

$$\Rightarrow \quad 0 = f(0) = S_f(0) = \frac{1}{2} - \sum_{K=1}^{\infty} \frac{4}{(2N-1)^2 \pi^2} \cdot \frac{\cos(0)}{1}$$

$$\Rightarrow \quad \frac{1}{\pi^2} \sum_{K=1}^{\infty} \frac{4}{(2N-1)^2} = \frac{1}{2} \Rightarrow \boxed{\sum_{K=1}^{\infty} \frac{1}{(2N-1)^2} = \frac{\pi^2}{8}}$$

$\boxed{P2}$ a) $a_0 = \int_{-1}^{1} x^2 dx = \frac{x^3}{3}\Big|_{-1}^{1} = \boxed{\frac{2}{3}} \Rightarrow \boxed{\frac{a_0}{2} = \frac{1}{3}}$

$$a_N = \int_{-1}^{1} x^2 \cdot \cos(N\pi x)\, dx \overset{\text{Par}}{=} 2\int_0^1 x^2 \cos(N\pi x)\, dx = 2\left[\cancel{x^2 \cdot \frac{\sin(N\pi x)}{N\pi}\Big|_0^1}^{\,0} - \int_0^1 \frac{2x \sin(N\pi x)}{N\pi} dx\right]$$

$$= \frac{-4}{N\pi}\left[x\left(\frac{-\cos(N\pi x)}{N\pi}\right)\Big|_0^1 + \int_0^1 \cancel{\frac{\cos(N\pi x)}{N\pi}}^{\,0} dx\right] = \boxed{\frac{4}{(N\pi)^2} \cdot (-1)^N}$$

$$b_N = \int_{-1}^{1} x^2 \cdot \sin(N\pi x)\, dx \overset{\text{Impar}}{=} 0$$

$$\Rightarrow \quad f(x) = \frac{1}{3} + \sum_{N=1}^{\infty} \frac{4}{(N\pi)^2} (-1)^N \cos(N\pi x)$$

Ver enlaces

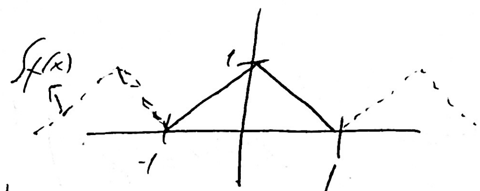
b) Como $S_f(x) \longrightarrow f(x)$

Usándolo para $\boxed{x=1} \Longrightarrow 1 = f(1) = S_f(1) = \frac{1}{3} + \sum_{N=1}^{\infty} \frac{4(-1)^N \cdot (-1)}{(N\pi)^2}$

$$\Longrightarrow \quad 1 = \frac{1}{3} + \frac{4}{\pi^2} \cdot \sum_{N=1}^{\infty} \frac{1}{N^2}$$

$$\Longrightarrow \quad \frac{2}{3} \cdot \frac{\pi^2}{4} = \sum_{N=1}^{\infty} \frac{1}{N^2} \Longrightarrow \boxed{\sum_{N=1}^{\infty} \frac{1}{N^2} = \frac{\pi^2}{6}}$$

$\boxed{P3}$ a) $f(x) = 1 - |x| \longrightarrow$



$$a_0 = \int_{-1}^{1} (1-|x|)\,dx \overset{Par}{=} 2\int_0^1 (1-x)\,dx = -(1-x)^2 \Big|_0^1 = \boxed{1}$$

$$\boxed{\frac{a_0}{2} = \frac{1}{2}}$$

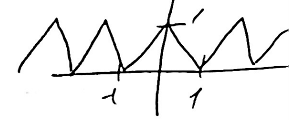
$$a_N = \int_{-1}^{1} (1-|x|) \operatorname{Cos}(N\pi x)\,dx \overset{Par}{=} 2\int_0^1 (1-x) \cdot \operatorname{Cos}(N\pi x)\,dx$$

$$= 2\left[\cancel{(1-x)\frac{\operatorname{Sin}(N\pi x)}{N\pi}\Big|_0^1}^{\,0} + \int_0^1 \frac{\operatorname{Sin}(N\pi x)}{N\pi}\,dx \right] = 2\left(\frac{-\operatorname{Cos}(N\pi x)}{(N\pi)^2}\right)\Big|_0^1$$

$$= \frac{2}{(N\pi)^2}\left[1 - (-1)^N\right] = \begin{cases} 0 & \text{Si } N \text{ es par} \\ \frac{4}{(N\pi)^2} & \text{Si } N \text{ es impar} \end{cases}$$

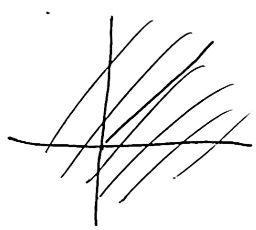
$$b_n = \int_{-1}^{1}(1-|x|)\sin(n\pi x)\,dx \overset{\text{Impar}}{=} 0$$

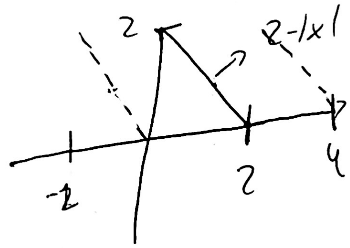
$$f(x) = \frac{1}{2} + \sum_{k=1}^{\infty}\frac{4}{(2k)^2\pi^2}\cos(n\pi x)$$ =



Como las función es par es equivalente a la serie de Cosenos

b)





Si $T=1$

$\Rightarrow$ Periodo $2T=2$

$$a_0 = \frac{1}{1}\int_{-1}^{1} f(x)\,dx \overset{\text{Periodicidad}}{=} \int_0^2 f(x)\,dx = \int_0^2 2-|x|\,dx = \int_0^2 (2-x)\,dx$$

$$= -\frac{(2-x)^2}{2}\Bigg|_0^2 = \boxed{2} \Rightarrow \boxed{\frac{a_0}{2} = 1}$$

$$a_n = \int_0^2 (2-x)\cdot\cos(n\pi x)\,dx = (2-x)\cdot\frac{\sin(n\pi x)}{n\pi}\Bigg|_0^2 + \int_0^2 \frac{\sin(n\pi x)}{n\pi}\,dx$$

$$= -\frac{\cos(n\pi x)}{(n\pi)^2}\Bigg|_0^2 = \boxed{0}$$

$$\boxed{\frac{2}{n\pi}}$$

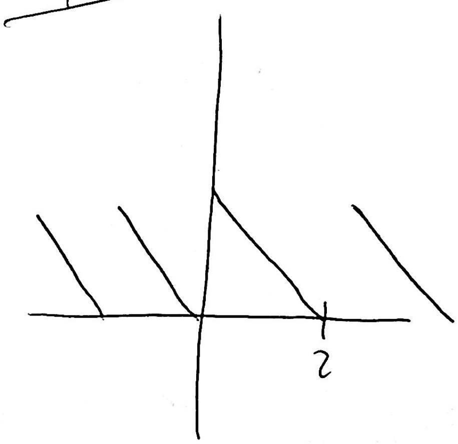
$$b_n = \int_0^2 (2-x)\sin(n\pi x)\,dx = (2-x)\cdot\left(\frac{-\cos(n\pi x)}{n\pi}\right)\Bigg|_0^2 - \int_0^2 \frac{\cos(n\pi x)}{n\pi}\,dx$$

$$b_n = \frac{2}{n\pi} - \cancel{\left.\frac{\sin(n\pi x)}{(n\pi)^2}\right|_0^2}^{\,0} = \boxed{\frac{2}{n\pi}}$$

$$\Rightarrow \boxed{S_f(x) = 1 + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cdot \sin(n\pi x)}$$

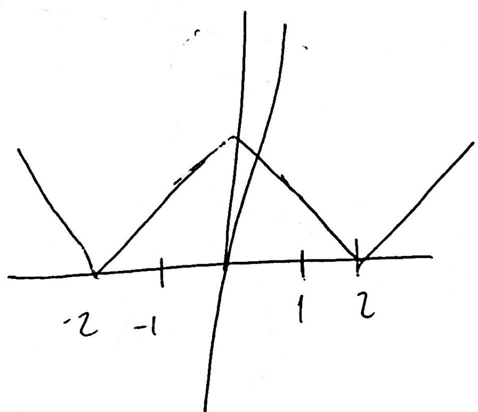
Comparación

Graficos de $S_f(x)$



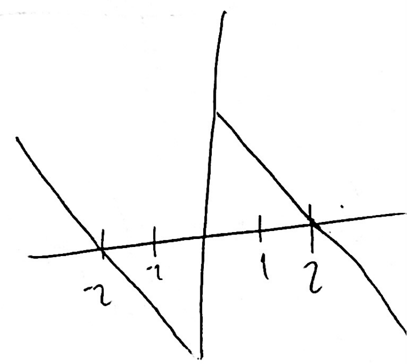
$f(x) = 2 - |x|$

Con $T = 1$ y $[0,2]$



$f(x) = 2 - |x|$

Con $T = 2$ y expansión par



$f(x) = 2 - |x|$

Con $T = 2$ y expansión impar