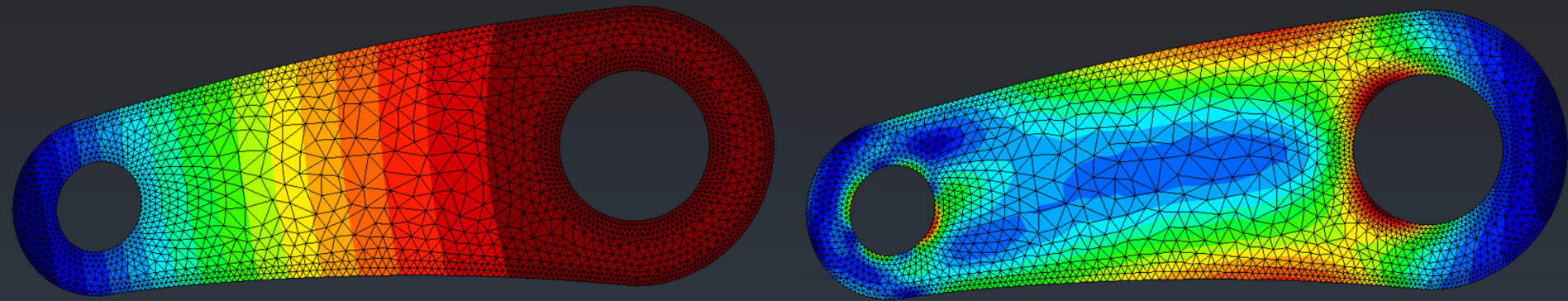


ELEMENTOS DE MÁQUINAS

ME5500



Alejandro Ortiz Bernardin

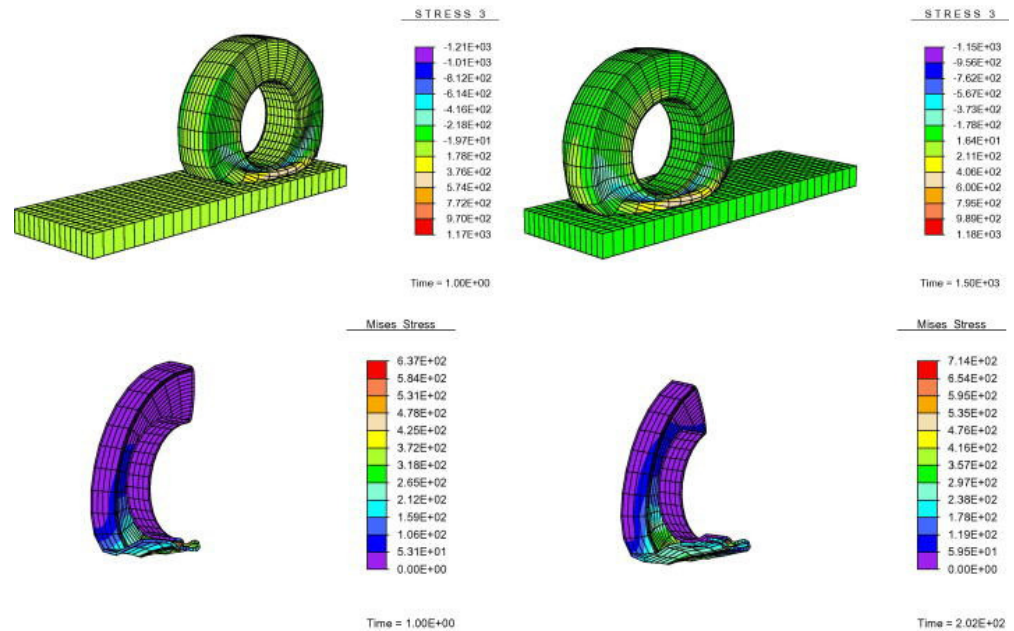
aortizb@uchile.cl

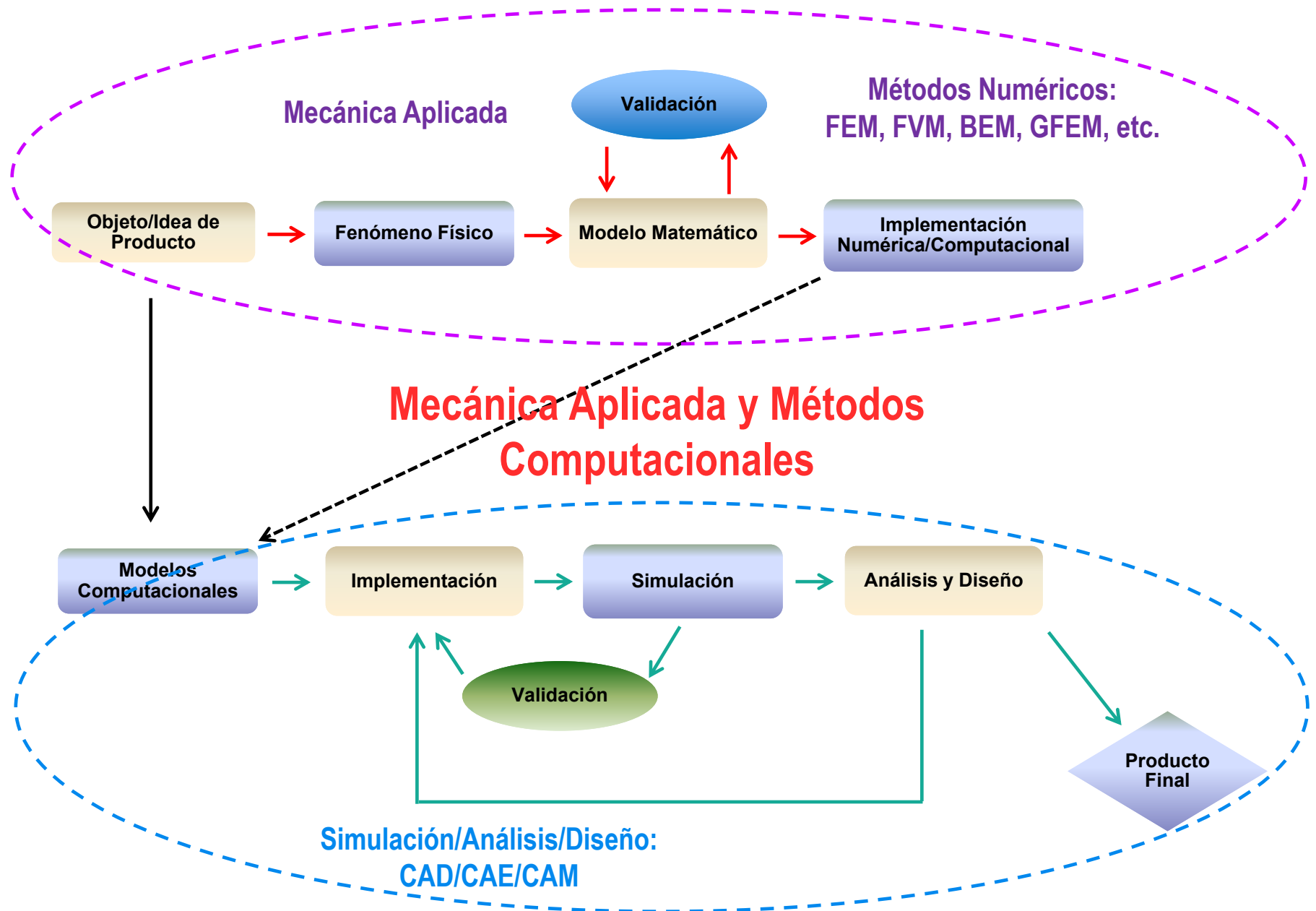
www.camlab.cl/alejandro

- I. Mecánica Computacional
- II. Modelación Mediante Método de Elementos Finitos
- III. Ejemplos Numéricos: Solución y Convergencia
- IV. Calidad del Mallado
- V. Singularidades (Concentración de Esfuerzos)
- VI. Principio de Saint Venant
- VII. Simetría
- VIII. Condiciones de Soporte
- IX. Elementos Basados en Líneas
- X. Elementos Basados en Superficies
- XI. Elementos Basados en Sólidos
- XII. Elemento Tipo Viga en ADINA
- XIII. Exactitud y Convergencia del Elemento Viga
- XIV. Elementos “Plate”/“Shell” en ADINA

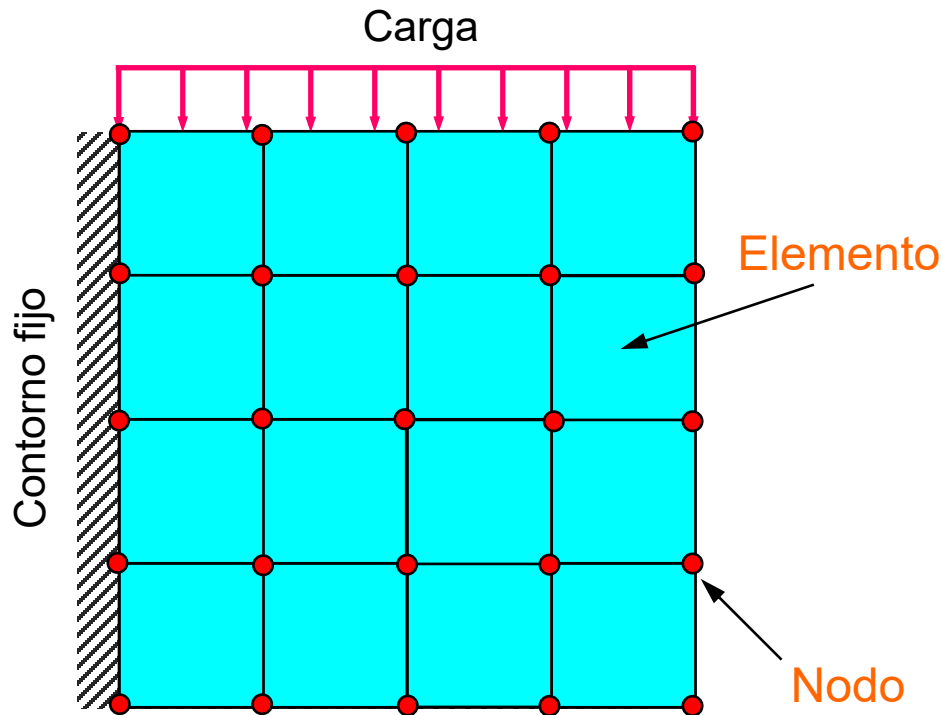
- Fenómeno físico
- Modelo matemático

- Modelo computacional
- Simulación computacional
- Análisis / Diseño





Modelación Mediante Elementos Finitos



Problema: obtener los esfuerzos y deformaciones en la placa.

- Método de aproximación
- Modelo geométrico
- Nodo
- Elemento
- Malla
- Discretización

Problema de Valor de Contorno (“Forma Fuerte”)

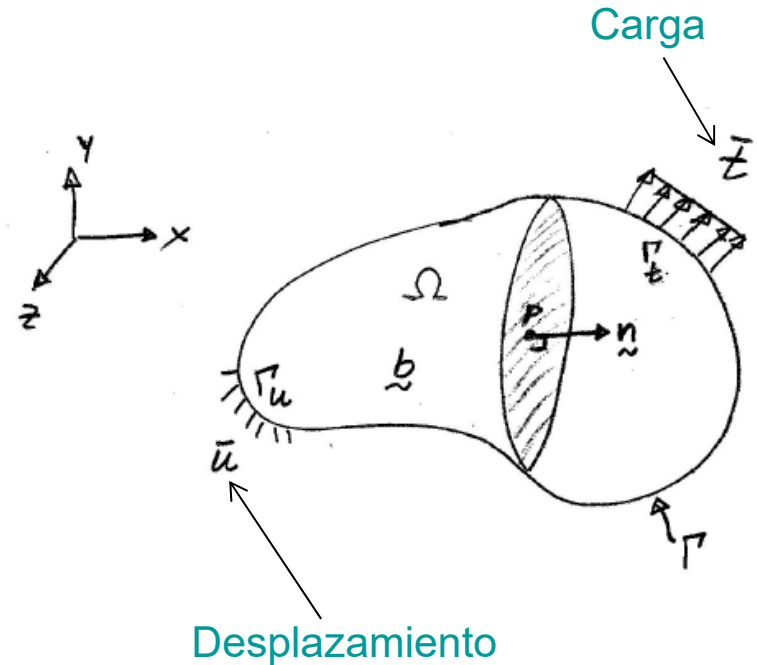
Esfuerzo Fuerza de cuerpo

$$\nabla \sigma + \mathbf{b} = 0 \quad \text{in } \Omega \quad \left. \vphantom{\nabla \sigma + \mathbf{b} = 0} \right\} \text{ EDP}$$
$$\sigma \cdot \mathbf{n} = \bar{\mathbf{t}} \quad \text{on } \Gamma_t \quad \left. \vphantom{\sigma \cdot \mathbf{n} = \bar{\mathbf{t}}} \right\} \text{ Condiciones de contorno}$$
$$\mathbf{u} = \bar{\mathbf{u}} \quad \text{on } \Gamma_u$$

Matriz del material

$$\sigma = \mathbf{D} \varepsilon \quad \left. \vphantom{\sigma = \mathbf{D} \varepsilon} \right\} \text{ Ley constitutiva}$$
$$\varepsilon = \nabla_s \mathbf{u} \quad \left. \vphantom{\varepsilon = \nabla_s \mathbf{u}} \right\} \text{ Compatibilidad cinemática}$$

Deformación



Validez del modelo

- Material con comportamiento lineal
- Pequeñas deformaciones ($\ll 0.2\%$)

Energía Potencial

$$\Pi = \frac{1}{2} \int_{\Omega} \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} d\Omega - \int_{\Omega} \mathbf{u}^T \mathbf{b} d\Omega - \int_{\Gamma_t} \mathbf{u}^T \bar{\mathbf{t}} d\Gamma$$

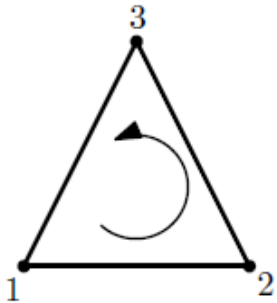
Energía Potencial Mínima (Trabajo Virtual) (“Forma débil”)

$$\delta\Pi = \int_{\Omega} \delta\boldsymbol{\varepsilon}^T \boldsymbol{\sigma} d\Omega - \int_{\Omega} \delta\mathbf{u}^T \mathbf{b} d\Omega - \int_{\Gamma_t} \delta\mathbf{u}^T \bar{\mathbf{t}} d\Gamma = 0$$

$$\delta\Pi = \int_{\Omega} \delta\boldsymbol{\varepsilon}^T \mathbf{D}\boldsymbol{\varepsilon} d\Omega - \int_{\Omega} \delta\mathbf{u}^T \mathbf{b} d\Omega - \int_{\Gamma_t} \delta\mathbf{u}^T \bar{\mathbf{t}} d\Gamma = 0$$

Discretización Mediante Elementos Finitos

Ejemplo: Discretización con elemento triangular de tres nodos



$$N_1 = \frac{1}{2A_e} \left(x_2^e y_3^e - x_3^e y_2^e + (y_2^e - y_3^e)x + (x_3^e - x_2^e)y \right)$$

$$N_2 = \frac{1}{2A_e} \left(x_3^e y_1^e - x_1^e y_3^e + (y_3^e - y_1^e)x + (x_1^e - x_3^e)y \right)$$

$$N_3 = \frac{1}{2A_e} \left(x_1^e y_2^e - x_2^e y_1^e + (y_1^e - y_2^e)x + (x_2^e - x_1^e)y \right)$$

Funciones de forma (polinomios lineales)

Discretización del desplazamiento

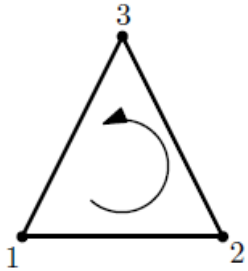
$$u_x^h = \sum_{a=1}^3 N_a u_{ax}, \quad u_y^h = \sum_{a=1}^3 N_a u_{ay}$$

(Interpolación lineal)

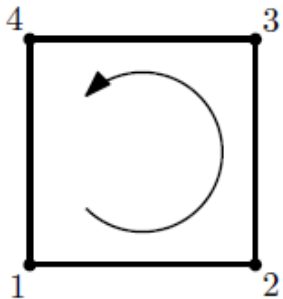
Desplazamiento nodal

Discretización Mediante Elementos Finitos

Discretización del desplazamiento (forma matricial)



$$\mathbf{u}^h = \begin{bmatrix} \sum_{a=1}^3 N_a u_{ax} \\ \sum_{a=1}^3 N_a u_{ay} \end{bmatrix} = \underbrace{\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 \end{bmatrix}}_{\mathbf{N}_e} \underbrace{\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{bmatrix}}_{\mathbf{u}_e}$$

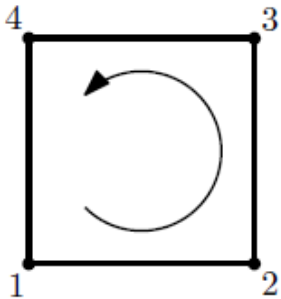


$$\mathbf{u}^h = \begin{bmatrix} \sum_{a=1}^4 N_a u_{ax} \\ \sum_{a=1}^4 N_a u_{ay} \end{bmatrix} = \underbrace{\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 & 0 \\ 0 & N_1 & 0 & N_2 & 0 & N_3 & 0 & N_4 \end{bmatrix}}_{\mathbf{N}_e} \underbrace{\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix}}_{\mathbf{u}_e}$$

Discretización Mediante Elementos Finitos

Discretización de la deformación

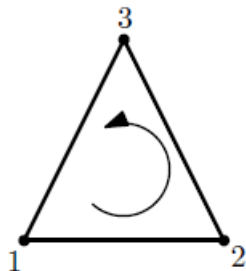
$$\varepsilon_{ij}^h = \frac{1}{2} \left(\frac{\partial u_i^h}{\partial x_j} + \frac{\partial u_j^h}{\partial x_i} \right)$$



$$\varepsilon^h = \begin{bmatrix} \varepsilon_{11}^h \\ \varepsilon_{22}^h \\ 2\varepsilon_{12}^h \end{bmatrix} = \begin{bmatrix} N_{1,x} & 0 & N_{2,x} & 0 & N_{3,x} & 0 & N_{4,x} & 0 \\ 0 & N_{1,y} & 0 & N_{2,y} & 0 & N_{3,y} & 0 & N_{4,y} \\ N_{1,y} & N_{1,x} & N_{2,y} & N_{2,x} & N_{3,y} & N_{3,x} & N_{4,y} & N_{4,x} \end{bmatrix}$$

B_e

$$\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \\ u_{4x} \\ u_{4y} \end{bmatrix} = \mathbf{u}_e$$



$$\varepsilon^h = \begin{bmatrix} \varepsilon_{11}^h \\ \varepsilon_{22}^h \\ 2\varepsilon_{12}^h \end{bmatrix} = \begin{bmatrix} N_{1,x} & 0 & N_{2,x} & 0 & N_{3,x} & 0 \\ 0 & N_{1,y} & 0 & N_{2,y} & 0 & N_{3,y} \\ N_{1,y} & N_{1,x} & N_{2,y} & N_{2,x} & N_{3,y} & N_{3,x} \end{bmatrix}$$

B_e

$$\begin{bmatrix} u_{1x} \\ u_{1y} \\ u_{2x} \\ u_{2y} \\ u_{3x} \\ u_{3y} \end{bmatrix} = \mathbf{u}_e$$

Matriz de Rigidez y Vector de Fuerzas Elementales

Forma débil:

$$\delta \Pi = \int_{\Omega} \delta \boldsymbol{\varepsilon}^T \mathbf{D} \boldsymbol{\varepsilon} d\Omega - \int_{\Omega} \delta \mathbf{u}^T \mathbf{b} d\Omega - \int_{\Gamma_t} \delta \mathbf{u}^T \bar{\mathbf{t}} d\Gamma = 0$$

Discretización (elemental): $\boldsymbol{\varepsilon}^h = \mathbf{B}_e \mathbf{u}_e$, $\mathbf{u}^h = \mathbf{N}_e \mathbf{u}_e$

$$\int_{\Omega_e} \delta \mathbf{u}_e^T \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e \mathbf{u}_e d\Omega_e - \int_{\Omega_e} \delta \mathbf{u}_e^T \mathbf{N}^T \mathbf{b} d\Omega_e - \int_{\Gamma_{te}} \delta \mathbf{u}_e^T \mathbf{N}_e^T \bar{\mathbf{t}} d\Gamma_e = 0$$

$$\delta \mathbf{u}_e^T \left\{ \int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e \mathbf{u}_e d\Omega_e - \int_{\Omega_e} \mathbf{N}^T \mathbf{b} d\Omega_e - \int_{\Gamma_{te}} \mathbf{N}_e^T \bar{\mathbf{t}} d\Gamma_e \right\} = 0$$

Debido a la arbitrariedad de los desplazamientos virtuales:

$$\underbrace{\left(\int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e d\Omega_e \right)}_{\mathbf{K}_e} \mathbf{u}_e = \underbrace{\int_{\Omega_e} \mathbf{N}^T \mathbf{b} d\Omega_e + \int_{\Gamma_{te}} \mathbf{N}_e^T \bar{\mathbf{t}} d\Gamma_e}_{\mathbf{f}_e} \quad \left. \vphantom{\int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e d\Omega_e} \right\} \text{Sistema elemental (local)}$$

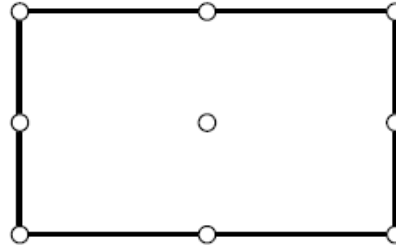
Elementos Finitos Sólidos Bidimensionales

Algunos elementos cuadriláteros/rectangulares

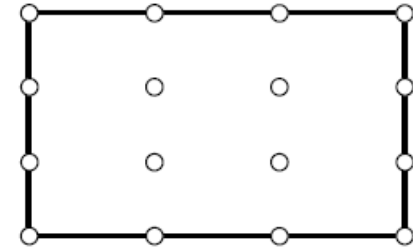
Lineal



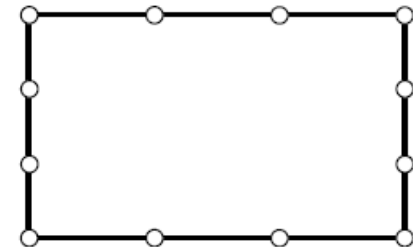
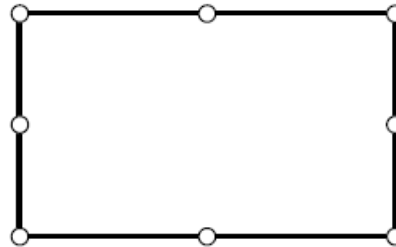
Cuadrático



Cúbico



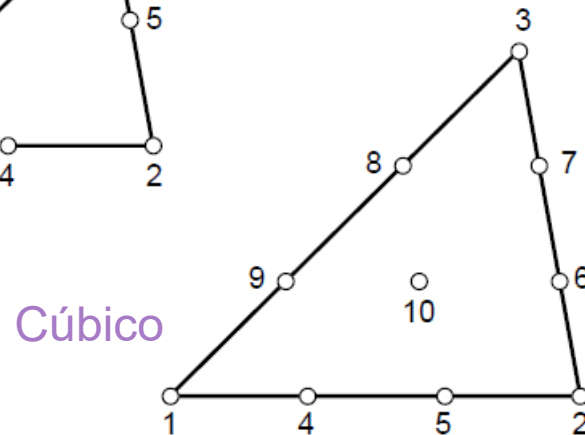
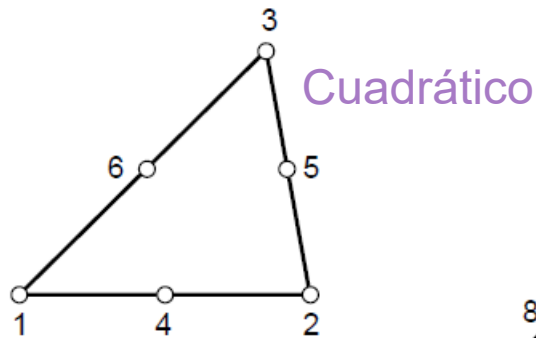
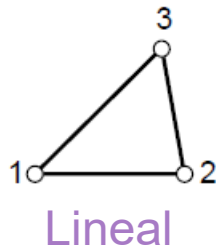
Lagrangianos



Serindípticos

Elementos Finitos Sólidos Bidimensionales

Algunos elementos triangulares



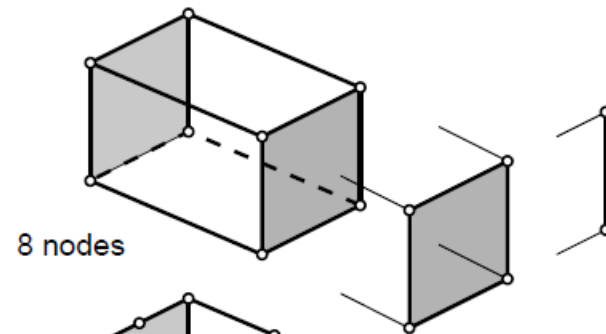
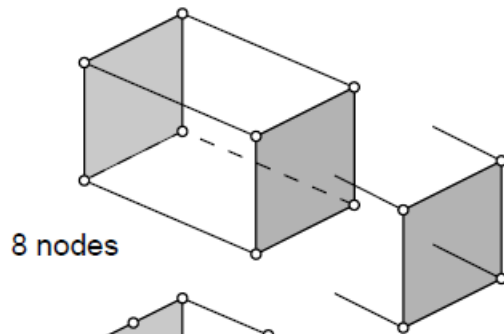
Elementos Finitos Sólidos Tridimensionales

Algunos elementos hexaédricos

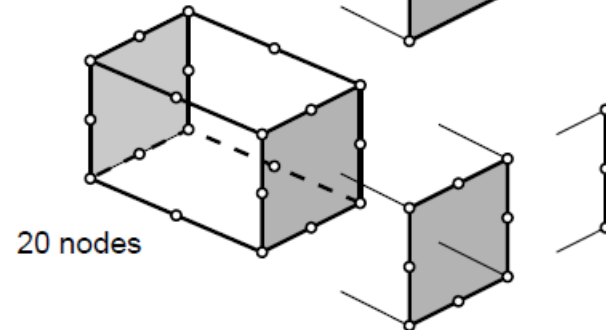
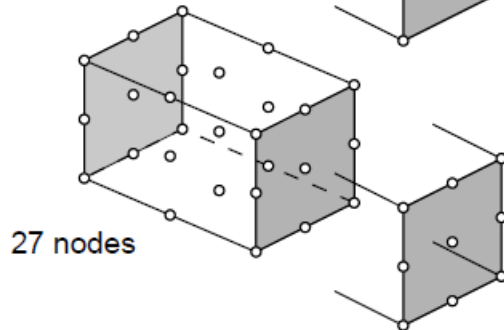
Lineal

Lagrangiano

Serindípito

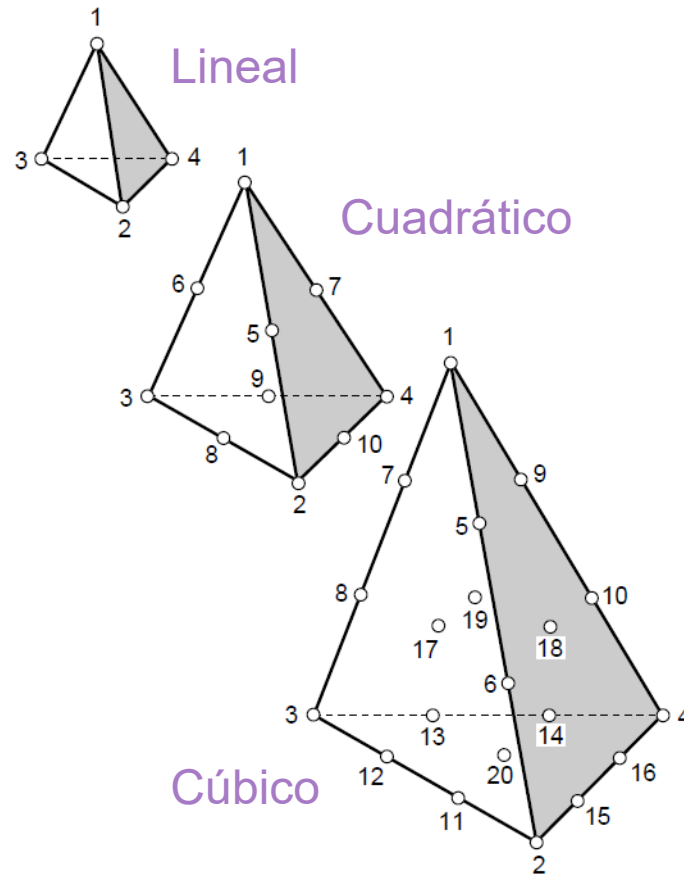


Cuadrático



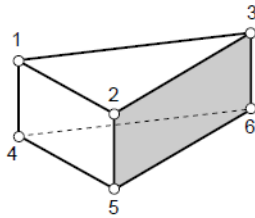
Elementos Finitos Sólidos Tridimensionales

Algunos elementos tetraédricos

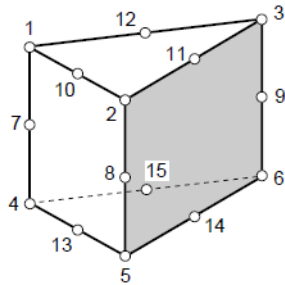


Elementos Finitos Sólidos Tridimensionales

Algunos elementos prismáticos

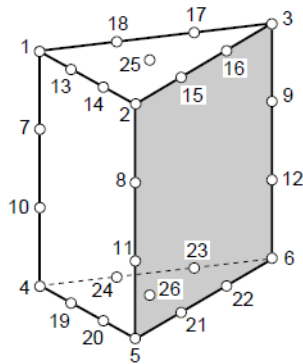


Lineal



Cuadrático

Elementos para transición
tetraédro - hexaédro



Cúbico

Algoritmo de Solución por Elementos Finitos

I. **Ensamble** de matrices y vectores elementales

Loop sobre elementos

$$\left(\begin{array}{c} \vdots \\ \vdots \\ \mathbf{K}_e \\ \vdots \\ \vdots \end{array} \right) \left(\begin{array}{c} \vdots \\ \vdots \\ u_e \\ \vdots \\ \vdots \end{array} \right) = \left(\begin{array}{c} \vdots \\ \vdots \\ f_e \\ \vdots \\ \vdots \end{array} \right)$$

Fin del loop

Sistema global

$$\mathbf{K} \mathbf{u} = \mathbf{f}$$

II. **Aplicación** de condiciones de contorno

III. **Solución** del sistema global de ecuaciones: $\mathbf{u} = \mathbf{K}^{-1} \mathbf{f}$

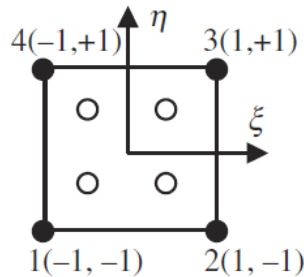
IV. **Post proceso** de deformaciones y esfuerzos

Cálculo Numérico de Esfuerzos y Deformaciones

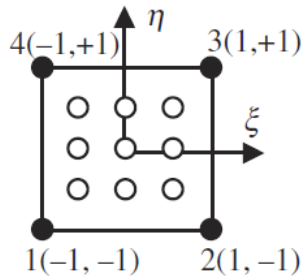
Integración Numérica

$$\begin{aligned}
 \mathbf{K}_e &= \int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e d\Omega_e = \int_{-1}^1 \int_{-1}^1 \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e |\mathbf{J}_e| d\xi d\eta \\
 &= \sum_{i=1}^2 \sum_{j=1}^2 w_i w_j |\mathbf{J}_e(\xi_i, \eta_j)| \underbrace{\mathbf{B}_e^T(\xi_i, \eta_j) \mathbf{D} \mathbf{B}_e(\xi_i, \eta_j)}_{\text{Esfuerzo/deformación en los puntos de integración}}
 \end{aligned}$$

Esfuerzo/deformación en los puntos de integración



(2 pts. por dirección)



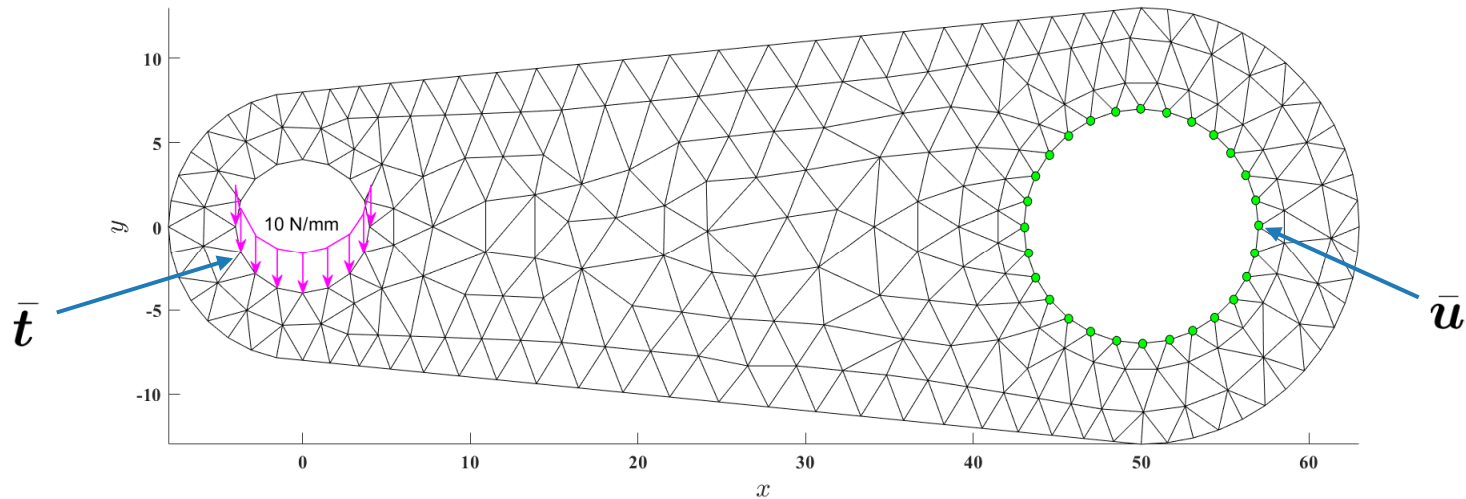
(3 pts. por dirección)

Coordenadas y pesos de puntos de integración.
Usar esta tabla para cada dirección.

m	ξ_j	w_j	Accuracy n
1	0	2	1
2	$-1/\sqrt{3}, 1/\sqrt{3}$	1,1	3
3	$-\sqrt{0.6}, 0, \sqrt{0.6}$	5/9, 8/9, 5/9	5
4	$-0.861136, -0.339981,$ $0.339981, 0.861136$	$0.347855, 0.652145,$ $0.652145, 0.347855$	7
5	$-0.906180, -0.538469, 0,$ $0.538469, 0.906180$	$0.236927, 0.478629, 0.568889,$ $0.478629, 0.236927$	9
6	$-0.932470, -0.661209, -0.238619,$ $0.238619, 0.661209, 0.932470$	$0.171324, 0.360762, 0.467914,$ $0.467914, 0.360762, 0.171324$	11

Ejemplo: Llave de Torque

Llave de torque de espesor unitario



Arreglos elementales:

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e d\Omega_e$$

$$\mathbf{f}_e = \int_{\Omega_e} \mathbf{N}^T \mathbf{b} d\Omega_e + \int_{\Gamma_{te}} \mathbf{N}_e^T \bar{\mathbf{t}} d\Gamma_e$$

Esfuerzo plano:

$$\mathbf{D} = \frac{E_Y}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$E_Y = 210 \text{ GPa}$$

$$\nu = 0.3$$

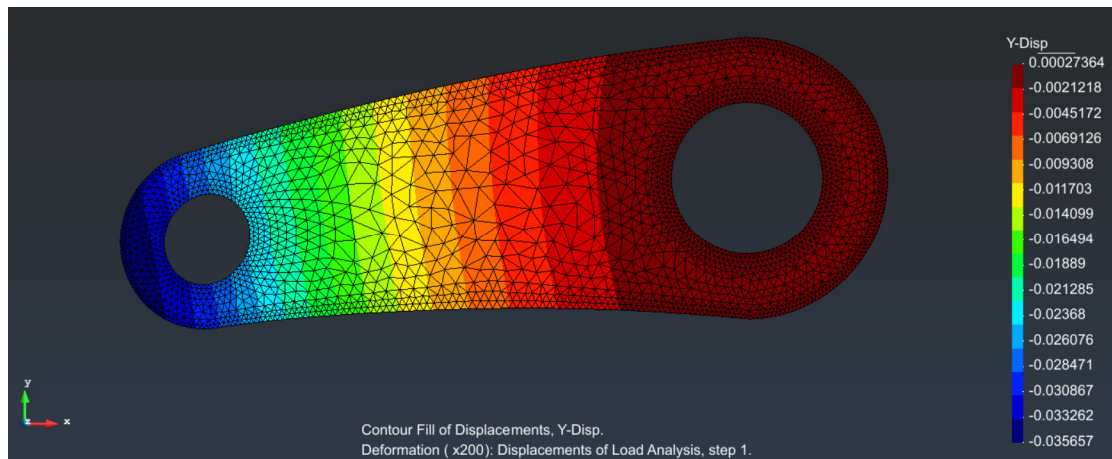
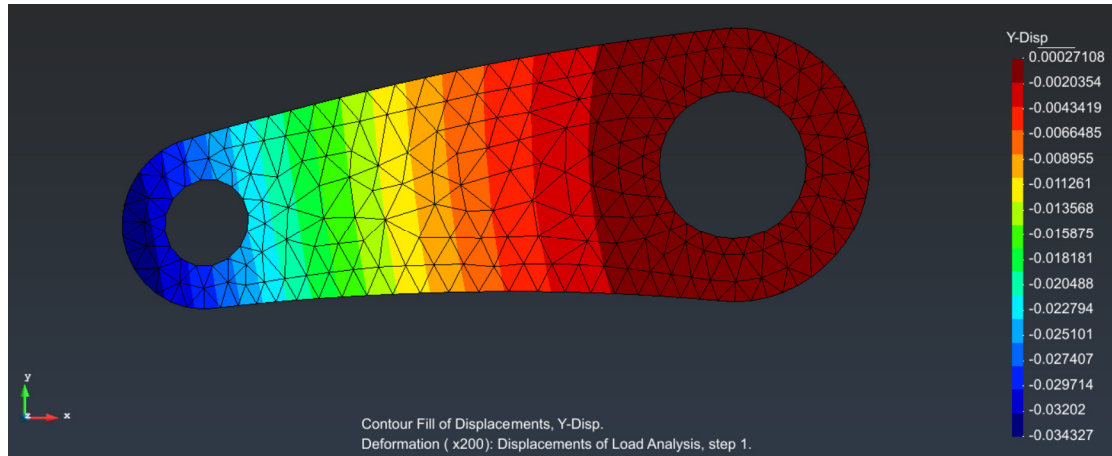
Condiciones de borde:

$$\bar{\mathbf{t}} = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

$$\bar{\mathbf{u}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Ejemplo: Llave de Torque

Efecto del refinamiento de malla en la solución

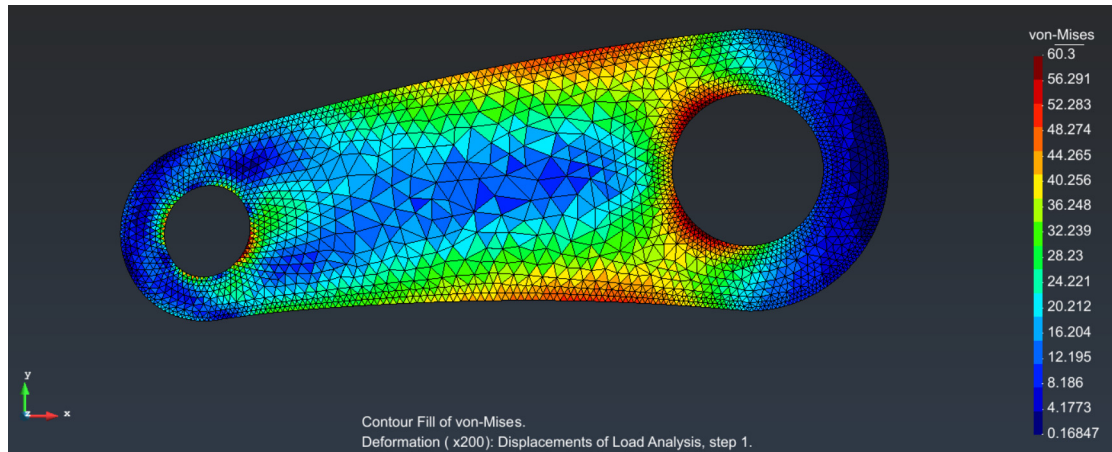
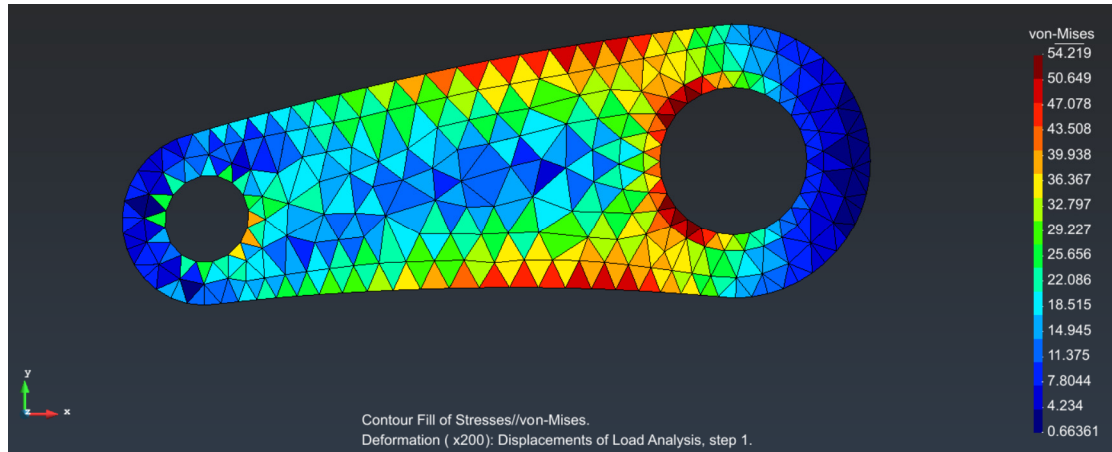


REFINAMIENTO

¿Es la solución independiente de la malla?

Ejemplo: Llave de Torque

Efecto del refinamiento de malla en la solución

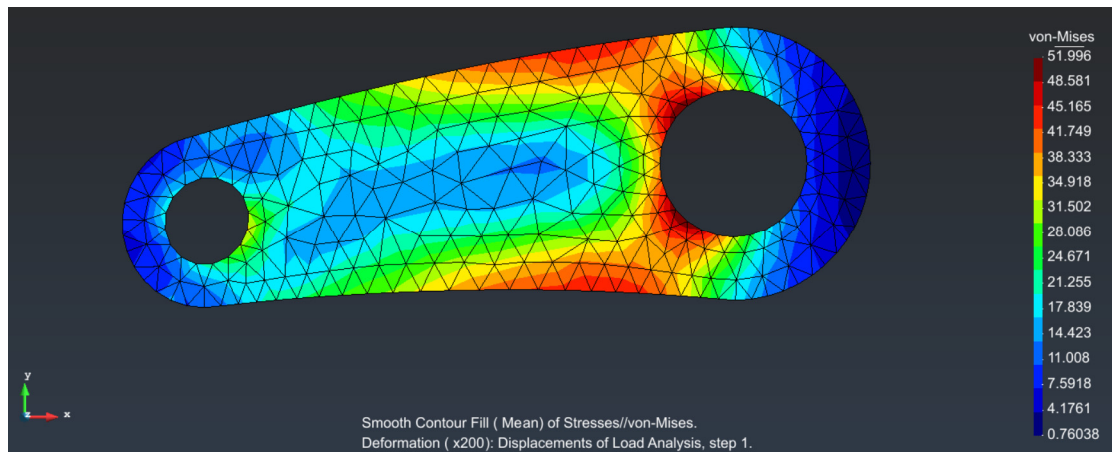
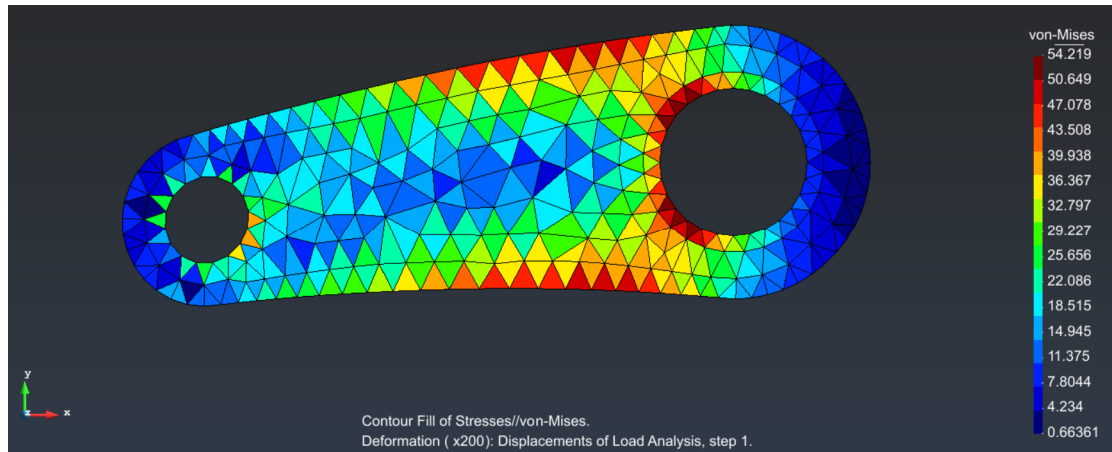


REFINAMIENTO

¿Es la solución independiente de la malla?

Ejemplo: Llave de Torque

Convergencia con respecto al alisado de esfuerzos (malla gruesa)

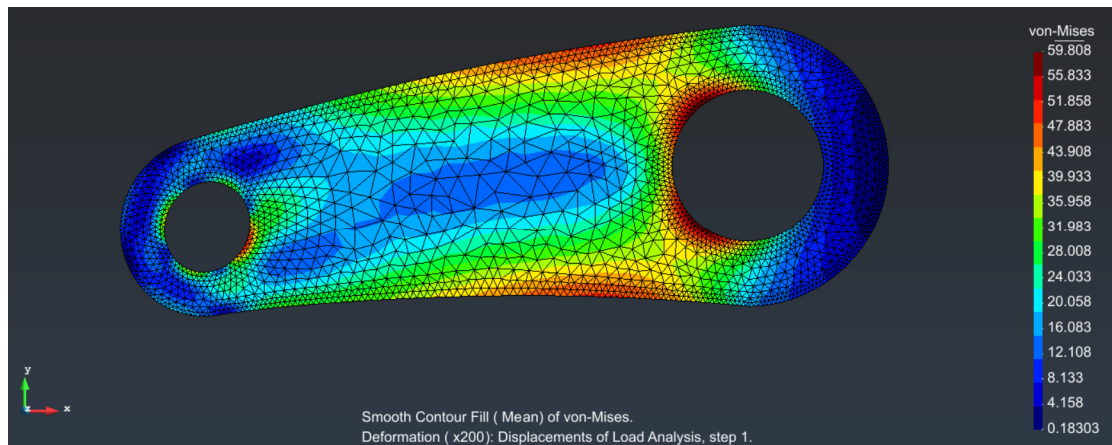
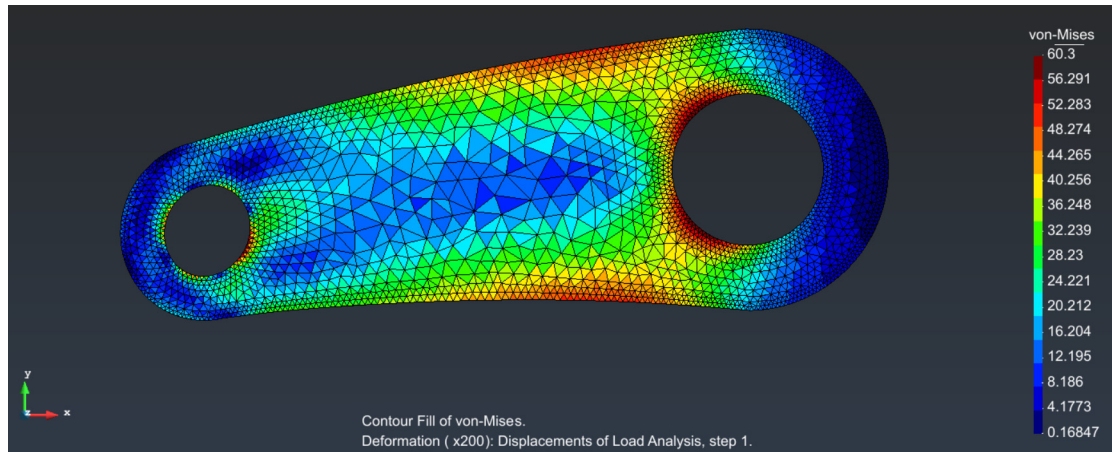


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Alisado:
$$\text{Error} = \frac{(54.22 - 51.99)}{54.22} \times 100 = 4.11\%$$

Ejemplo: Llave de Torque

Convergencia con respecto al alisado de esfuerzos (malla fina)

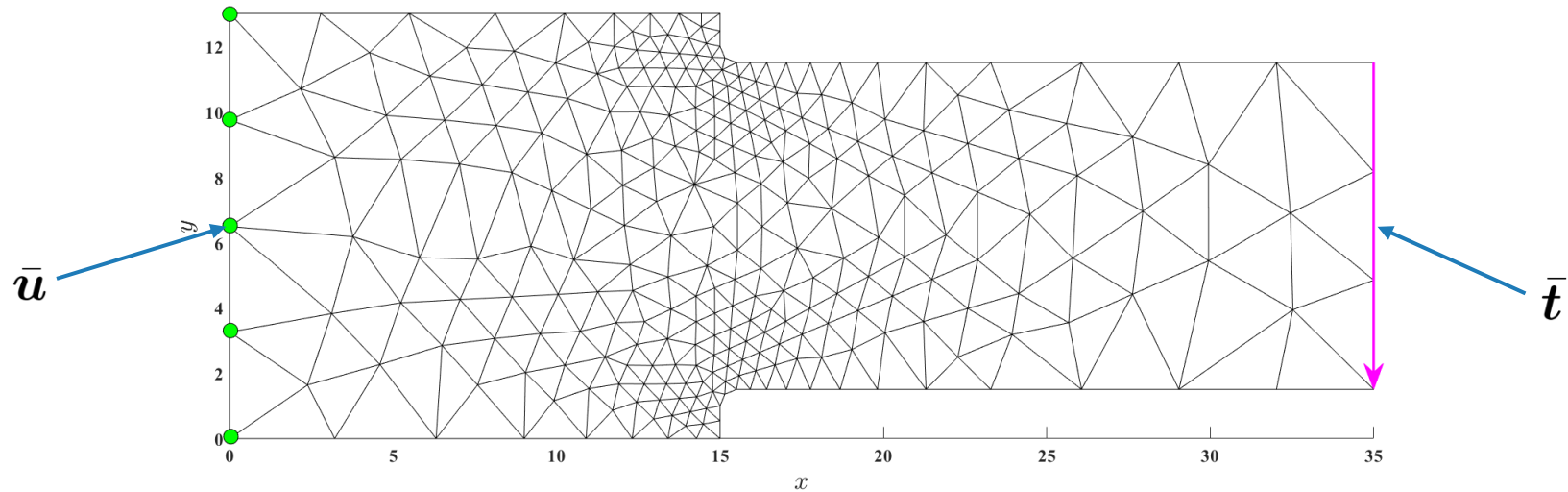


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Alisado:
$$\text{Error} = \frac{(60.30 - 59.81)}{60.30} \times 100 = 0.81\%$$

Ejemplo: Flexión de una Placa con Filetes

Placa de espesor unitario



Arreglos elementales:

$$\mathbf{K}_e = \int_{\Omega_e} \mathbf{B}_e^T \mathbf{D} \mathbf{B}_e d\Omega_e$$

$$\mathbf{f}_e = \int_{\Omega_e} \mathbf{N}^T \mathbf{b} d\Omega_e + \int_{\Gamma_{te}} \mathbf{N}_e^T \bar{\mathbf{t}} d\Gamma_e$$

Esfuerzo plano:

$$\mathbf{D} = \frac{E_Y}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$E_Y = 210 \text{ GPa}$$

$$\nu = 0.3$$

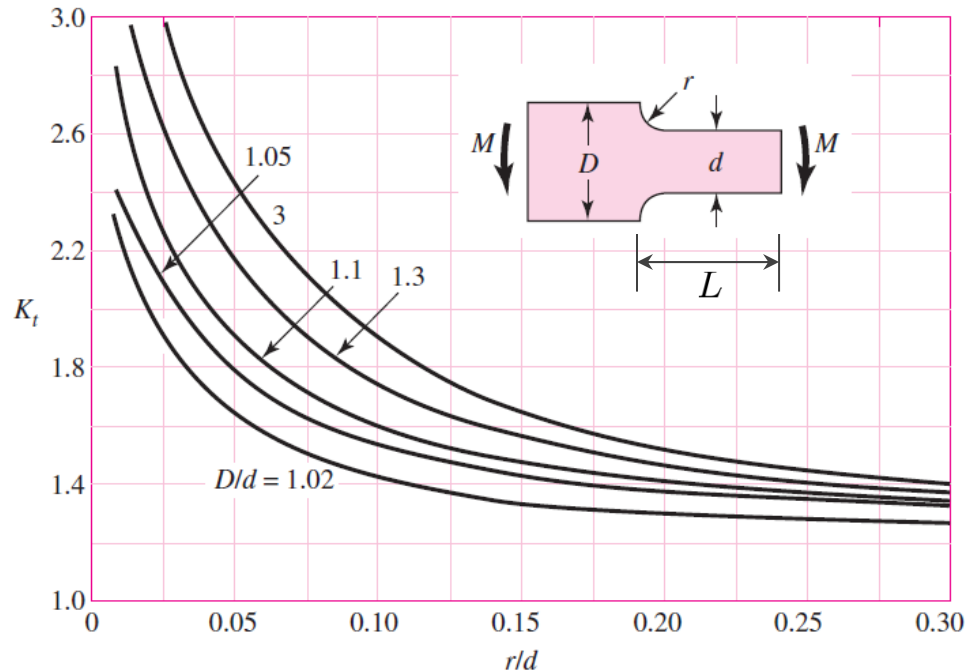
Condiciones de borde:

$$\bar{\mathbf{t}} = \begin{bmatrix} 0 \\ -10 \end{bmatrix}$$

$$\bar{\mathbf{u}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Ejemplo: Flexión de una Placa con Filetes

Solución en base a pruebas fotoelásticas



$$\begin{aligned} M &= |\bar{t}_y| \times d \times L \\ &= 10 \times 10 \times 20 \\ &= 2000 \text{ Nmm} \end{aligned}$$

$$\begin{aligned} \sigma_0 &= \frac{Md/2}{I} \\ &= 120 \text{ MPa} \end{aligned}$$

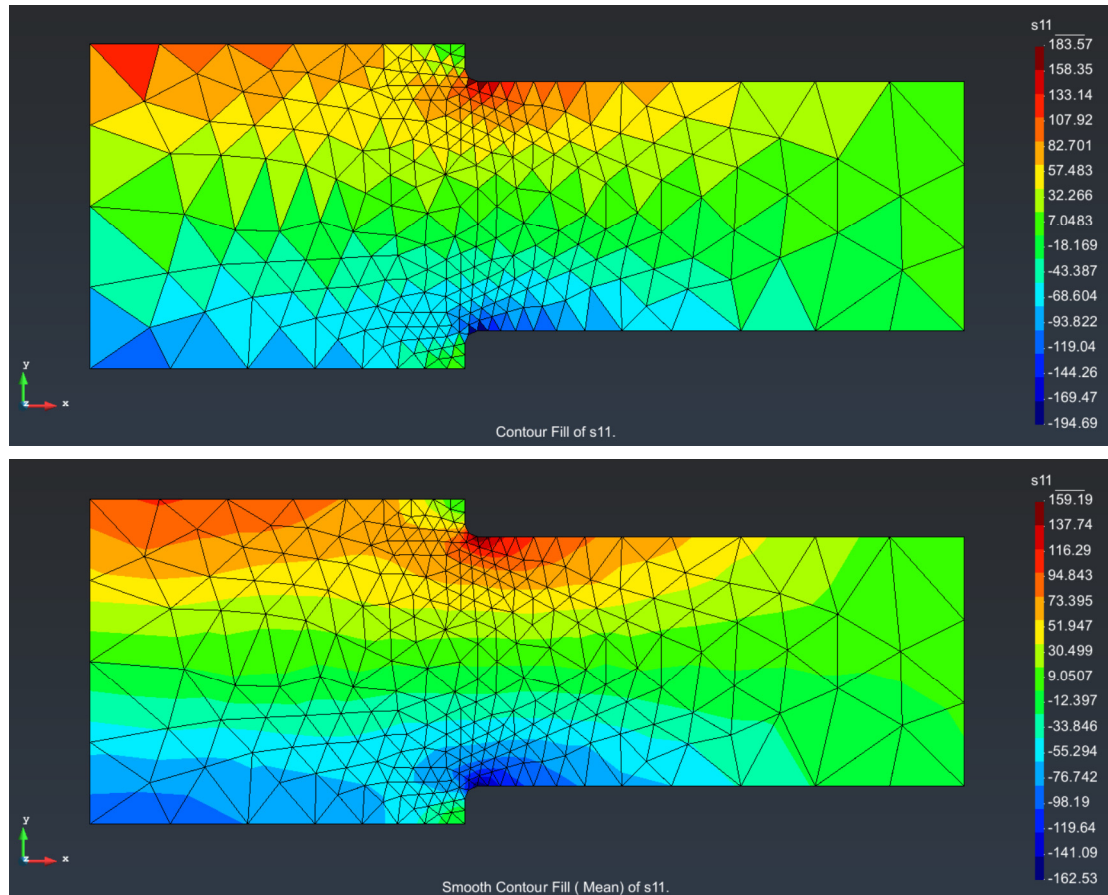
$$\begin{aligned} r/d &= 0.5/10 = 0.05 \\ D/d &= 13/10 = 1.3 \end{aligned}$$

$$\longrightarrow K_t \approx 2.18 \longrightarrow$$

$$\begin{aligned} \sigma_{\max} &= K_t \sigma_0 \\ &= 261.6 \text{ MPa} \end{aligned}$$

Ejemplo: Flexión de una Placa con Filetes

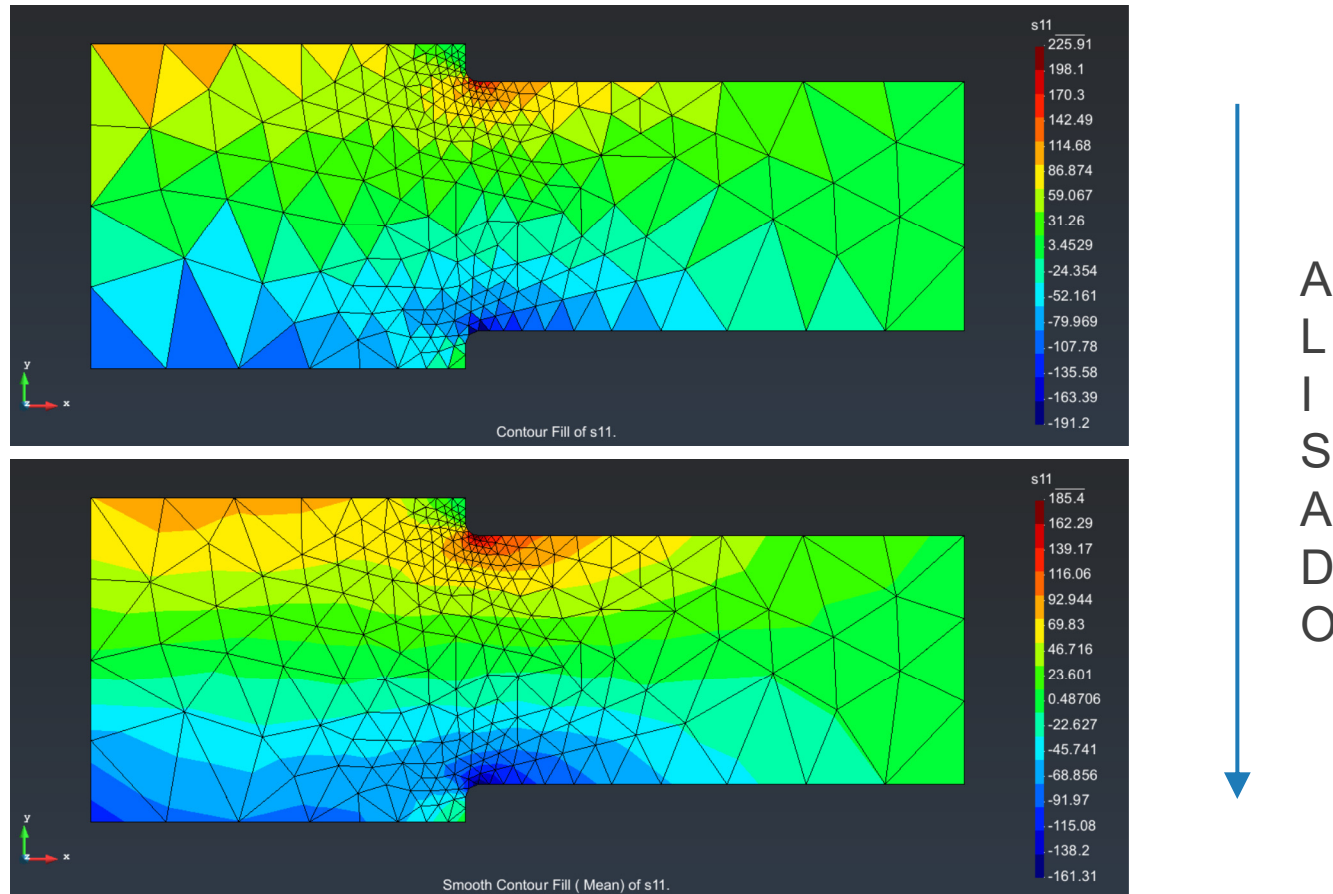
Convergencia con respecto al alisado de esfuerzos (malla gruesa)



Alisado: $\text{Error} = \frac{(183.57 - 159.19)}{183.57} \times 100 = 13.28\%$

Ejemplo: Flexión de una Placa con Filetes

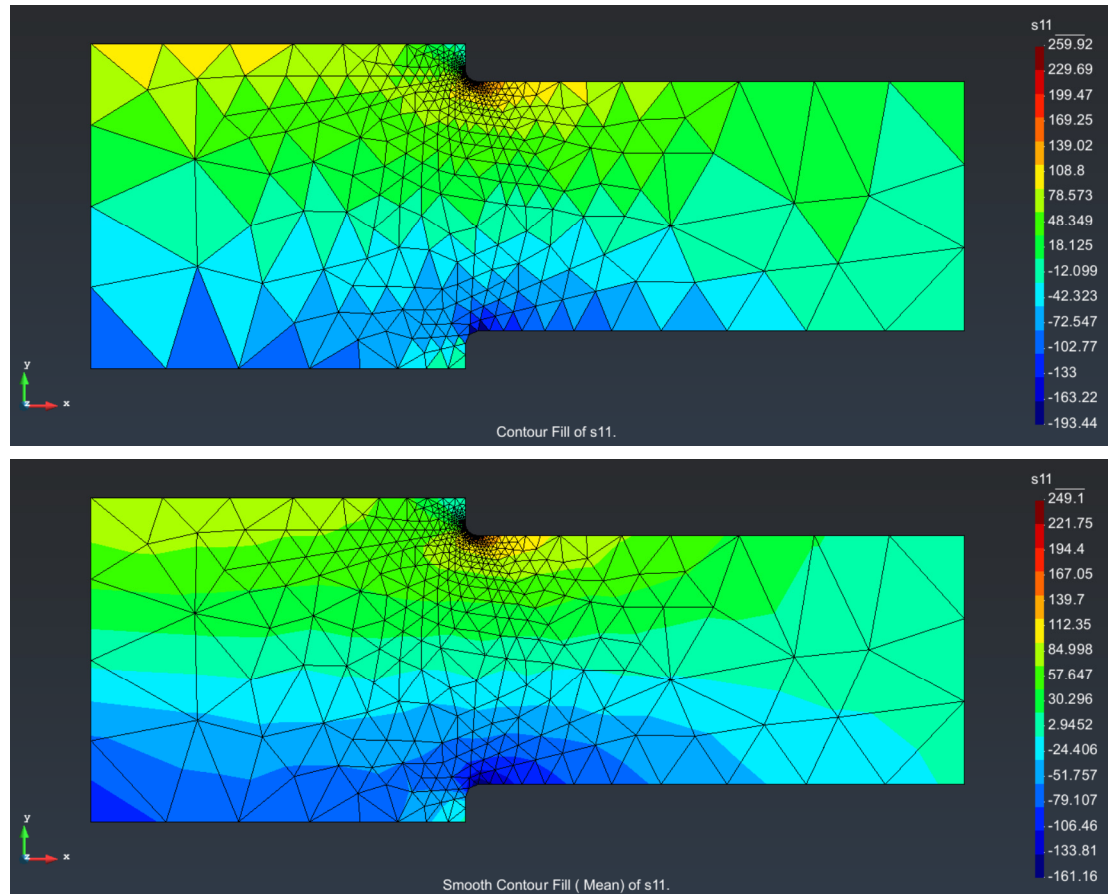
Convergencia con respecto al alisado de esfuerzos (**malla media**)



Alisado:
$$\text{Error} = \frac{(225.91 - 185.4)}{225.91} \times 100 = 17.93\%$$

Ejemplo: Flexión de una Placa con Filetes

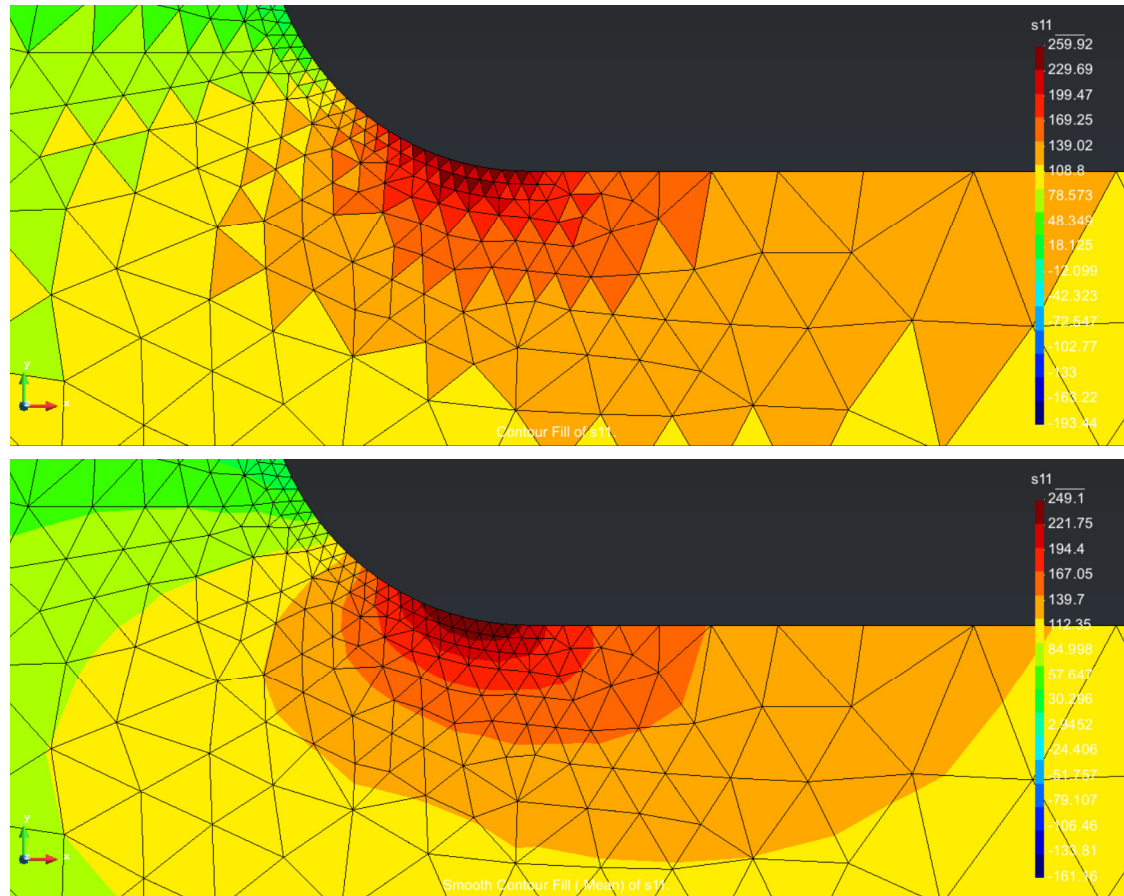
Convergencia con respecto al alisado de esfuerzos (**malla fina**)



Alisado:
$$\text{Error} = \frac{(259.92 - 249.1)}{259.92} \times 100 = 4.16\%$$

Ejemplo: Flexión de una Placa con Filetes

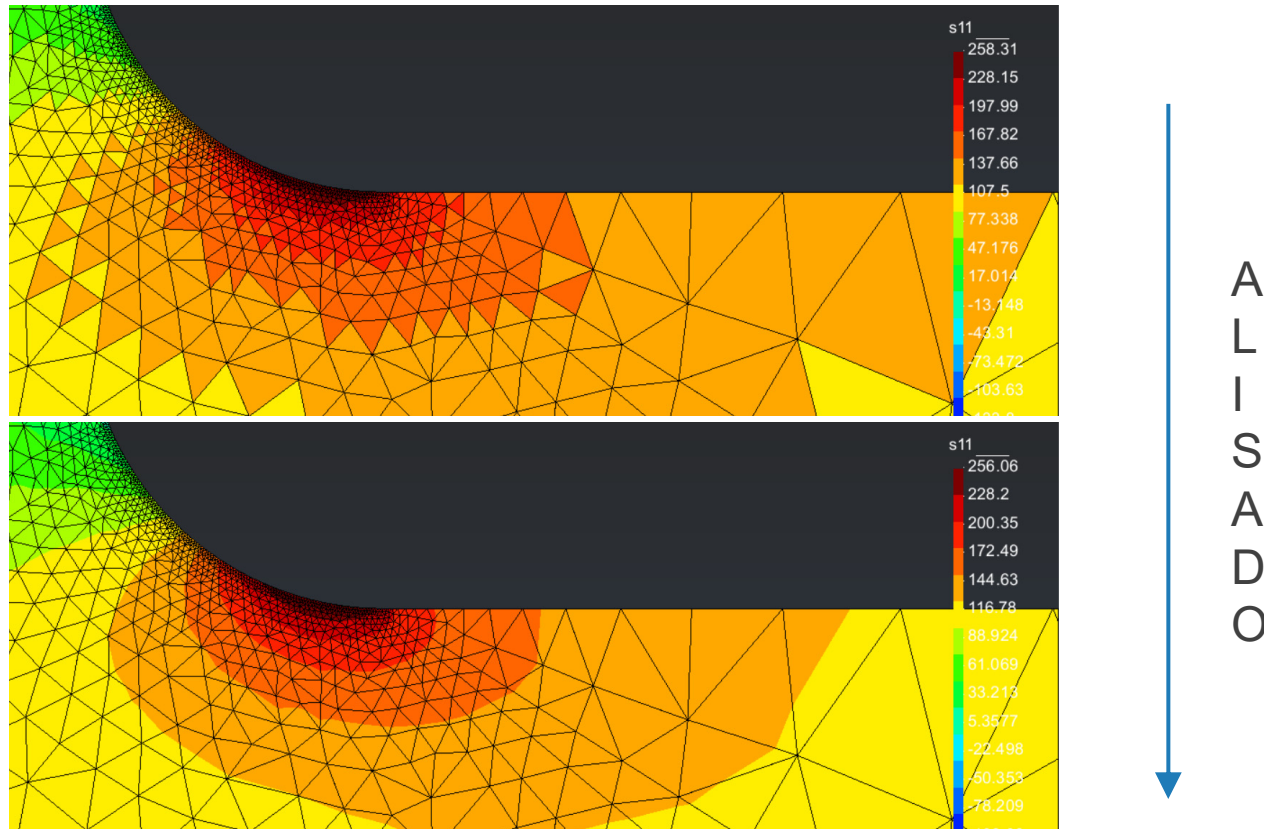
Convergencia con respecto a la solución exacta (malla fina)



Solución:
$$\text{Error} = \frac{(261.6 - 249.1)}{261.6} \times 100 = 4.77\%$$

Ejemplo: Flexión de una Placa con Filetes

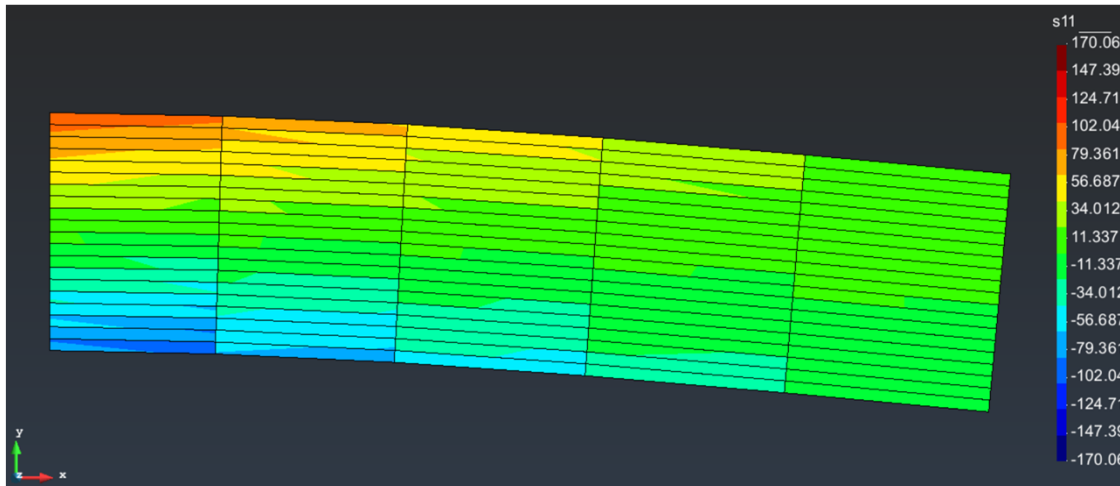
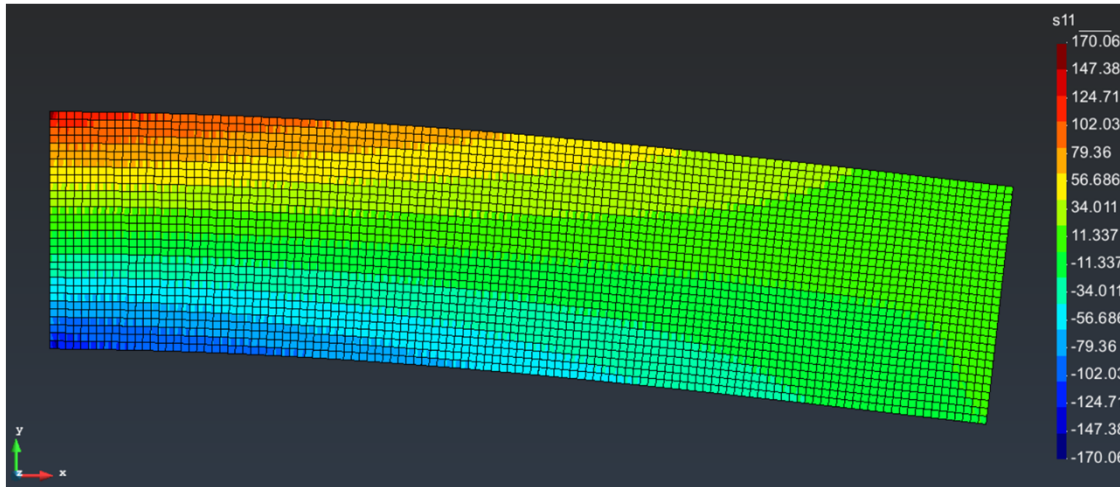
Convergencia con respecto al alisado y solución (malla ultra fina)



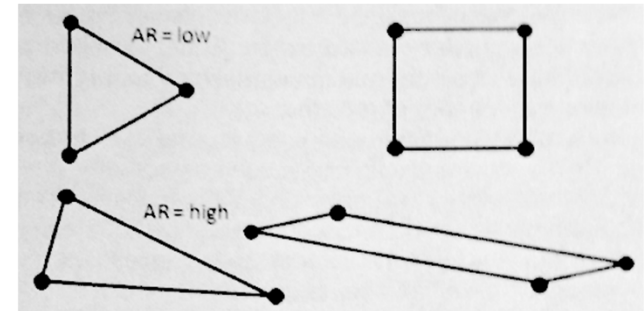
Alisado: $\text{Error} = \frac{(258.31 - 256.06)}{258.31} \times 100 = 0.87\%$

Solución: $\text{Error} = \frac{(261.6 - 256.06)}{261.6} \times 100 = 2.12\%$

Calidad de Mallado: Razón de Aspecto

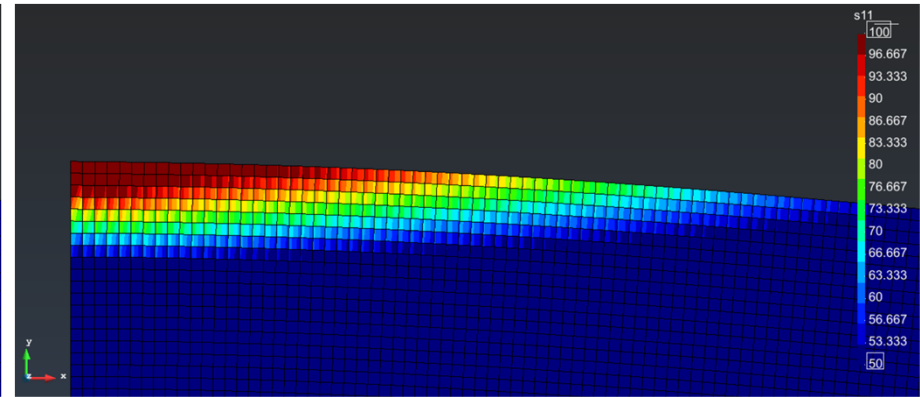
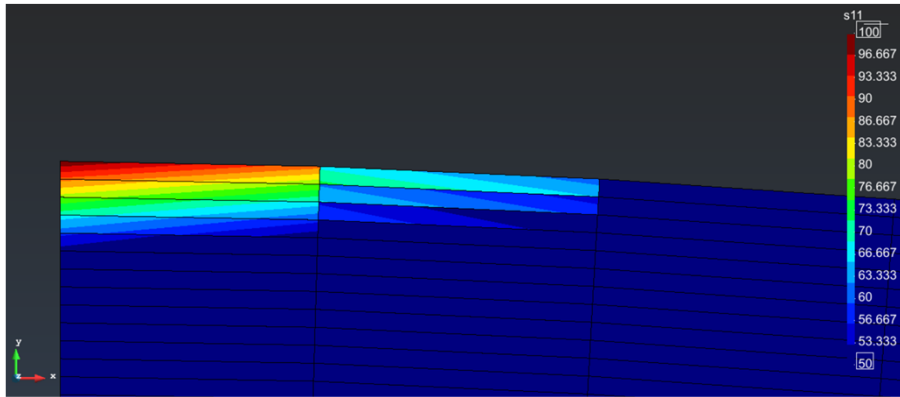


- La **razón de aspecto** (AR) debe ser baja para producir buenos resultados en esfuerzo y deformación

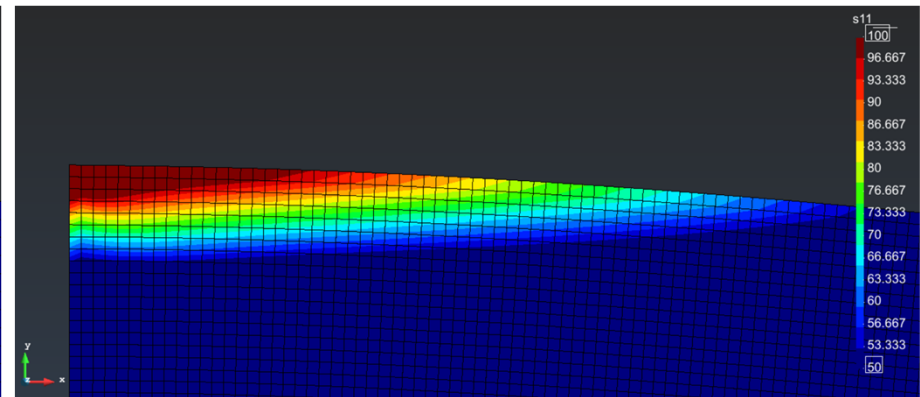
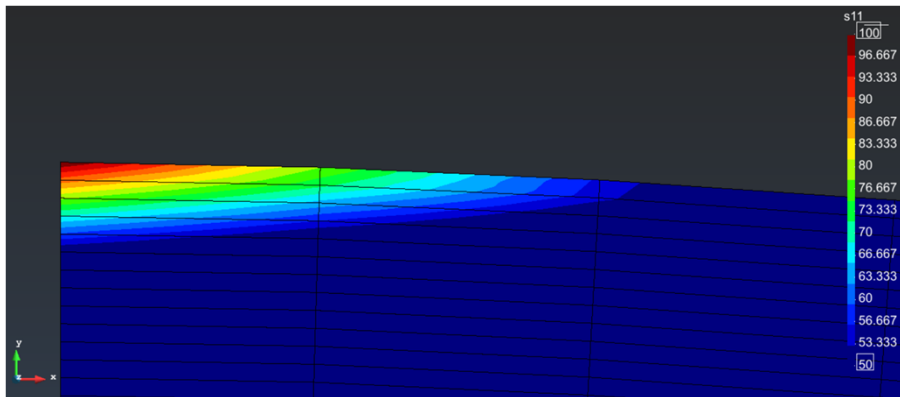


- Una **razón de aspecto** alta no captura correctamente el campo de esfuerzos y deformaciones

Calidad de Mallado: Razón de Aspecto

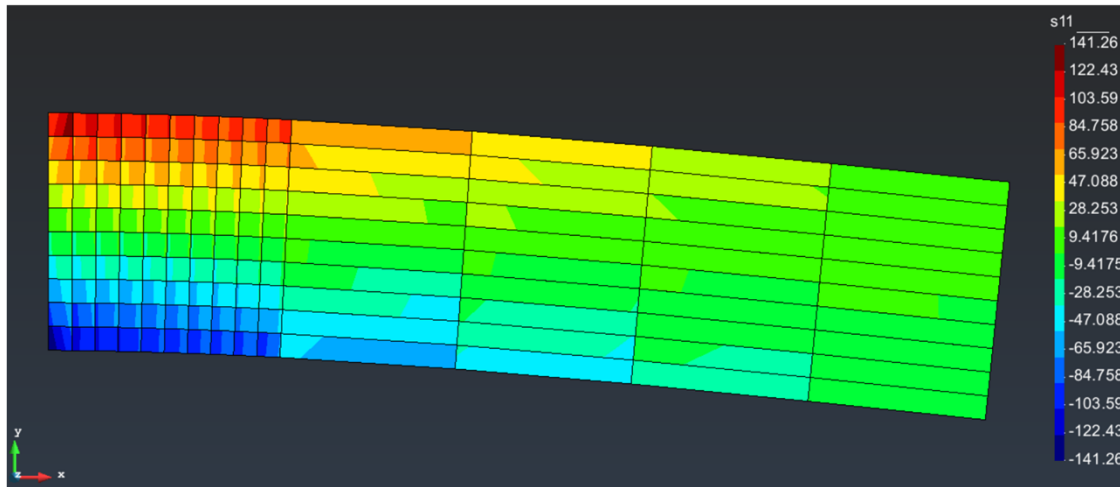


Alisado de esfuerzos

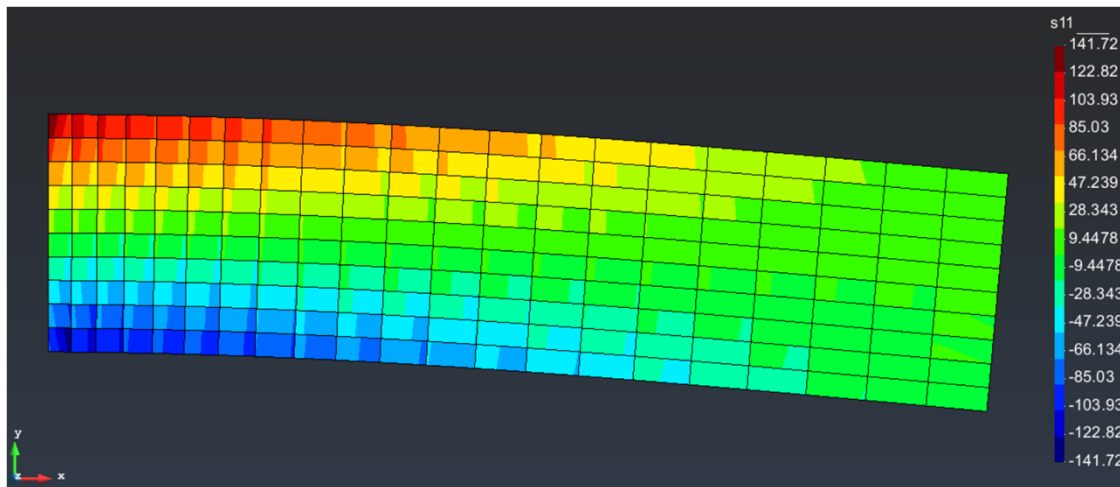


Una **razón de aspecto alta** no captura correctamente el campo de esfuerzos y deformaciones

Calidad del Mallado: Transición de Elementos



- Esfuerzos y deformaciones pueden verse afectados por la transición de elementos en la malla

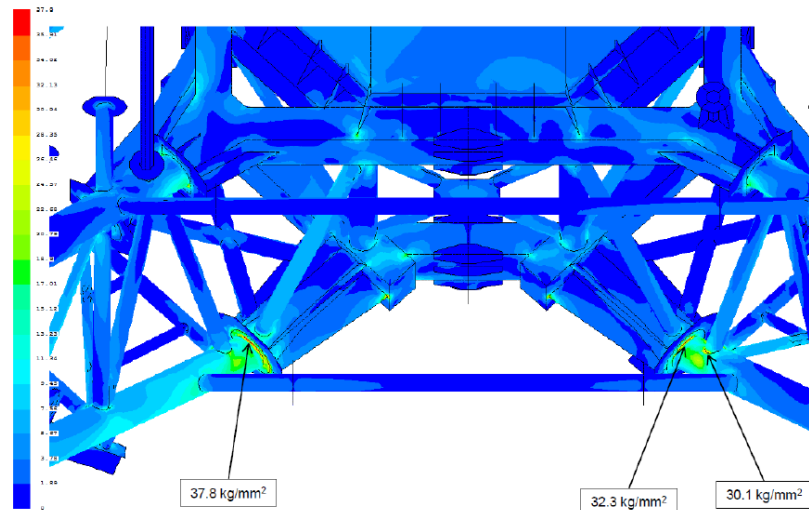


- Una transición suave desde elementos gruesos a finos, o viceversa, produce mejores resultados

Singularidades (Concentración de Esfuerzos)

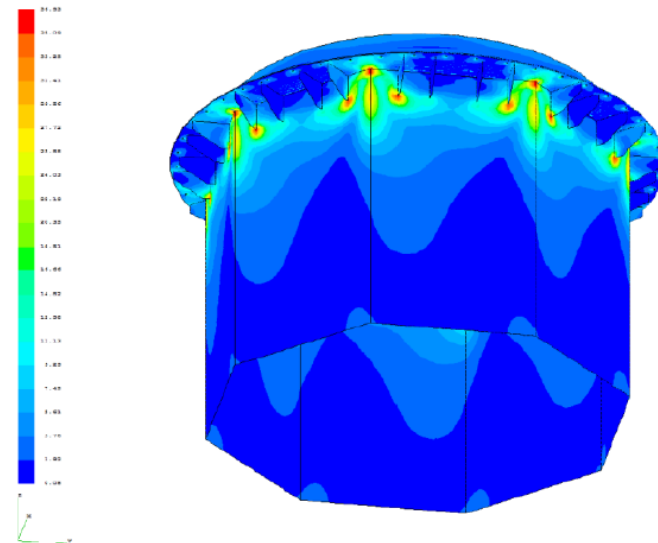
Causas

- Geometría
- Ley del material

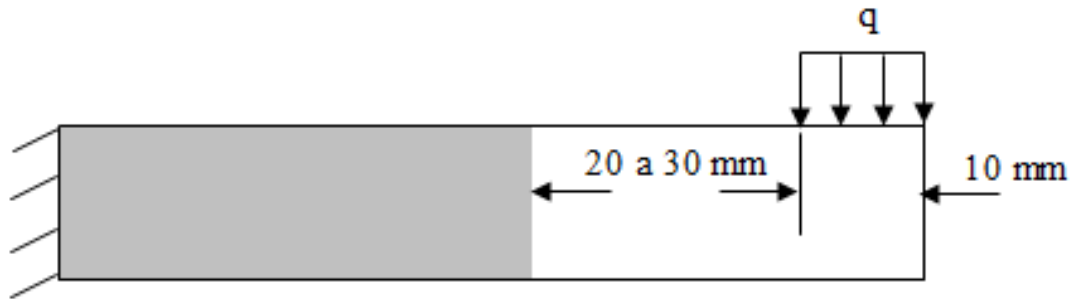


Potenciales concentradores

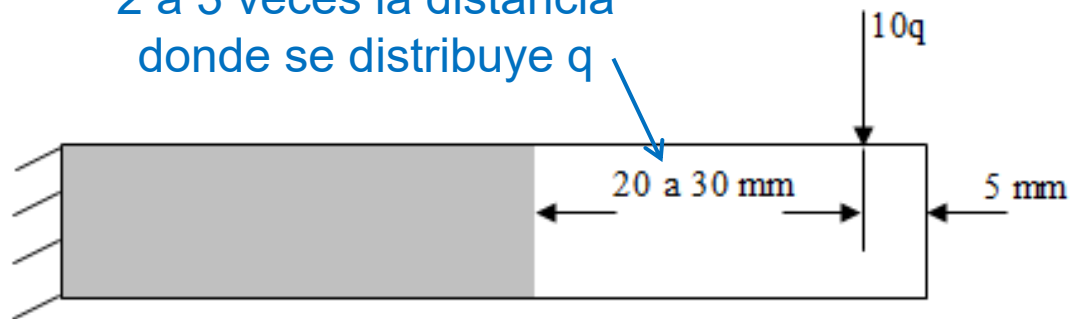
- Cantos vivos
- Esquinas (con o sin filete)
- Discontinuidad del material



Principio de Saint Venant



2 a 3 veces la distancia
donde se distribuye q



Zonas sombreadas
poseen estados de
esfuerzos similares

¿Cómo puede utilizarse este principio en una zona de concentración de esfuerzos ficticia?

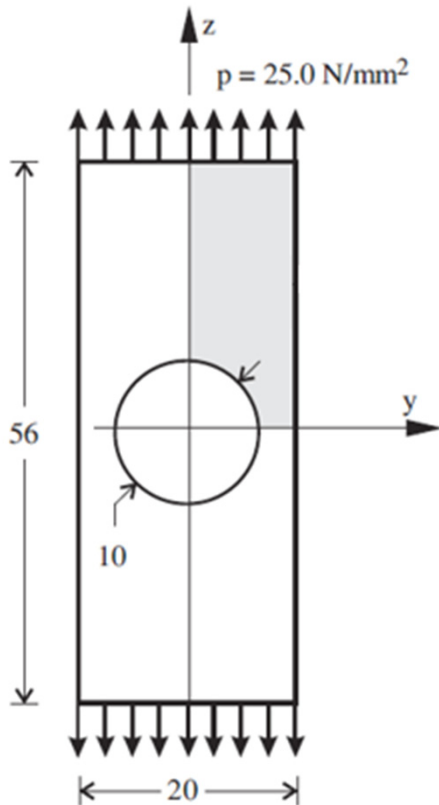


Figura A-15-1

Barra en tensión o compresión simple con un agujero transversal. $\sigma_0 = F/A$, donde $A = (w - d)t$ y t es el espesor.

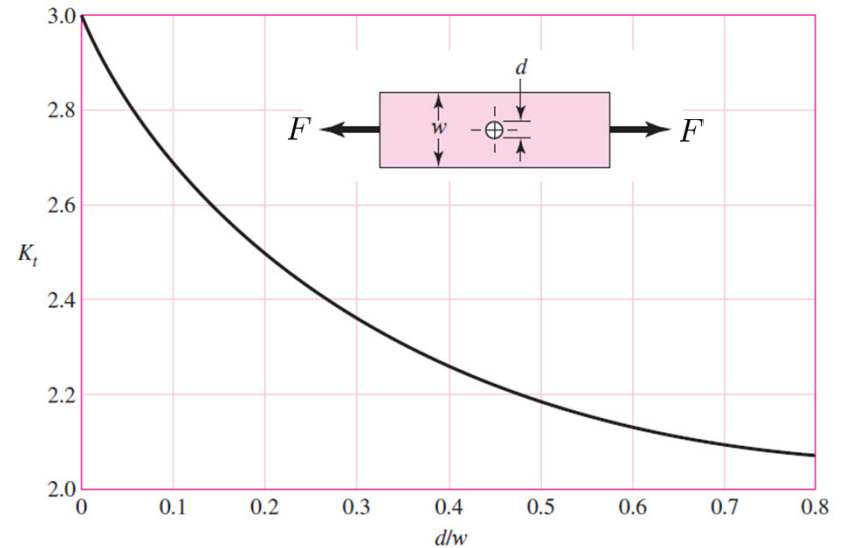
$$\begin{aligned}\sigma_0 &= \frac{F}{(w - d)t} \\ &= \frac{25 \times 20}{(20 - 10)1} \\ &= 50 \text{ MPa}\end{aligned}$$

$$E_Y = 210 \text{ GPa}$$

$$\nu = 0.3$$

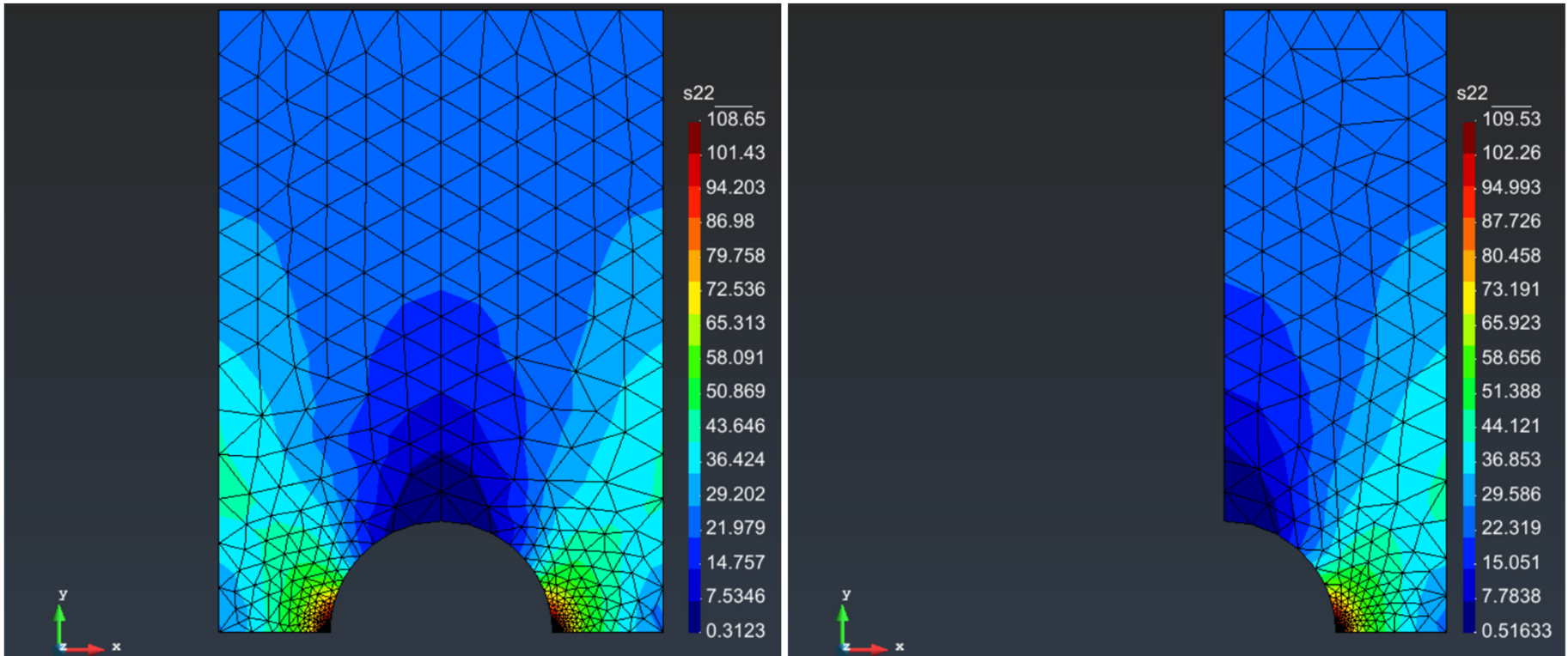
Espesor unitario

$$\begin{aligned}\sigma_{\max} &= K_t \sigma_0 \\ &= 109 \text{ MPa}\end{aligned}$$

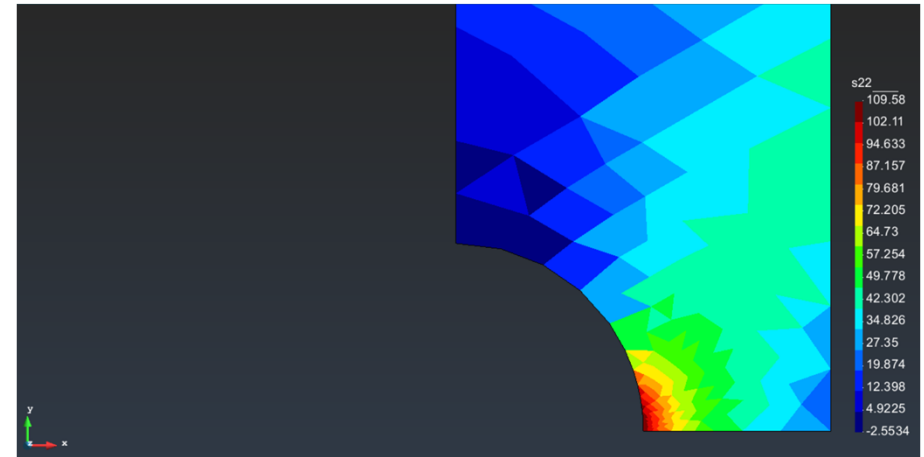
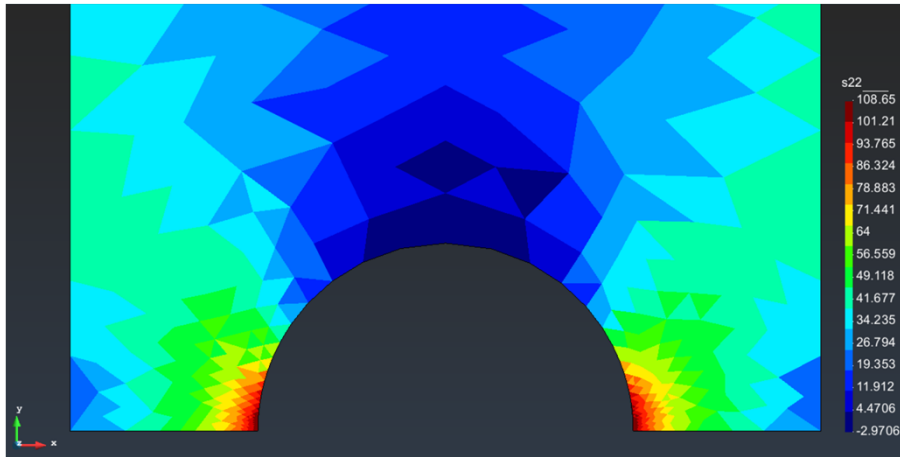


$$d/w = 10/20 = 0.5$$

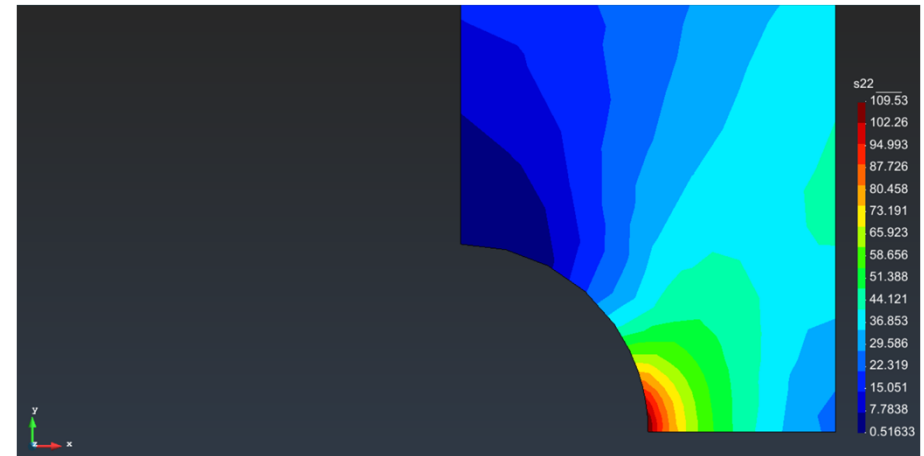
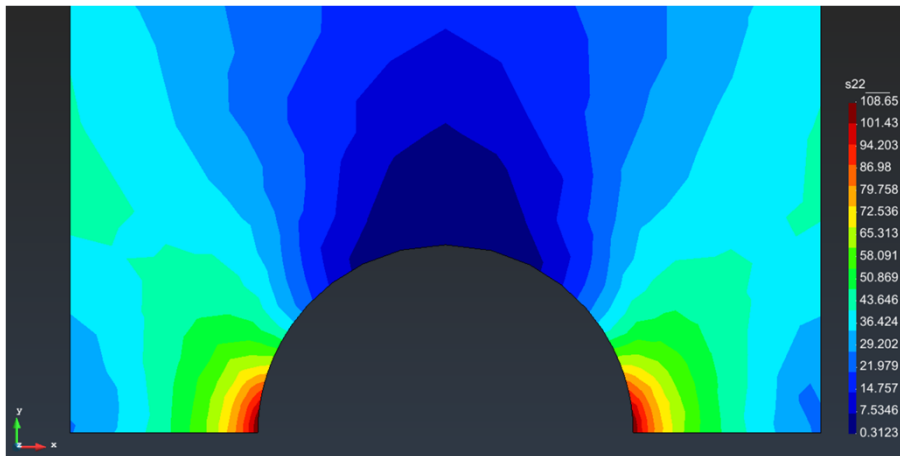
$$K_t \approx 2.18$$



Campo de esfuerzos es coincidente



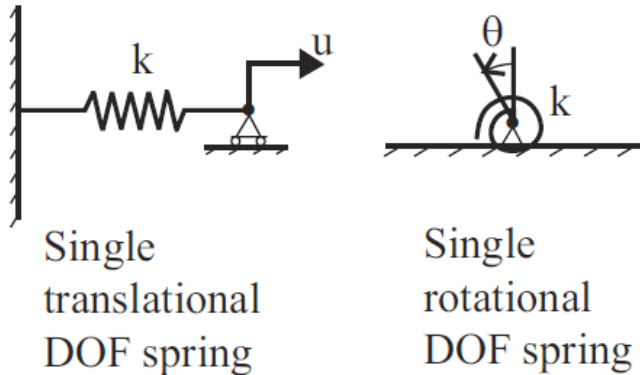
Alisado de esfuerzos



- Desplazamientos prescritos

$$u_i = f(t)$$

- “Spring elements”



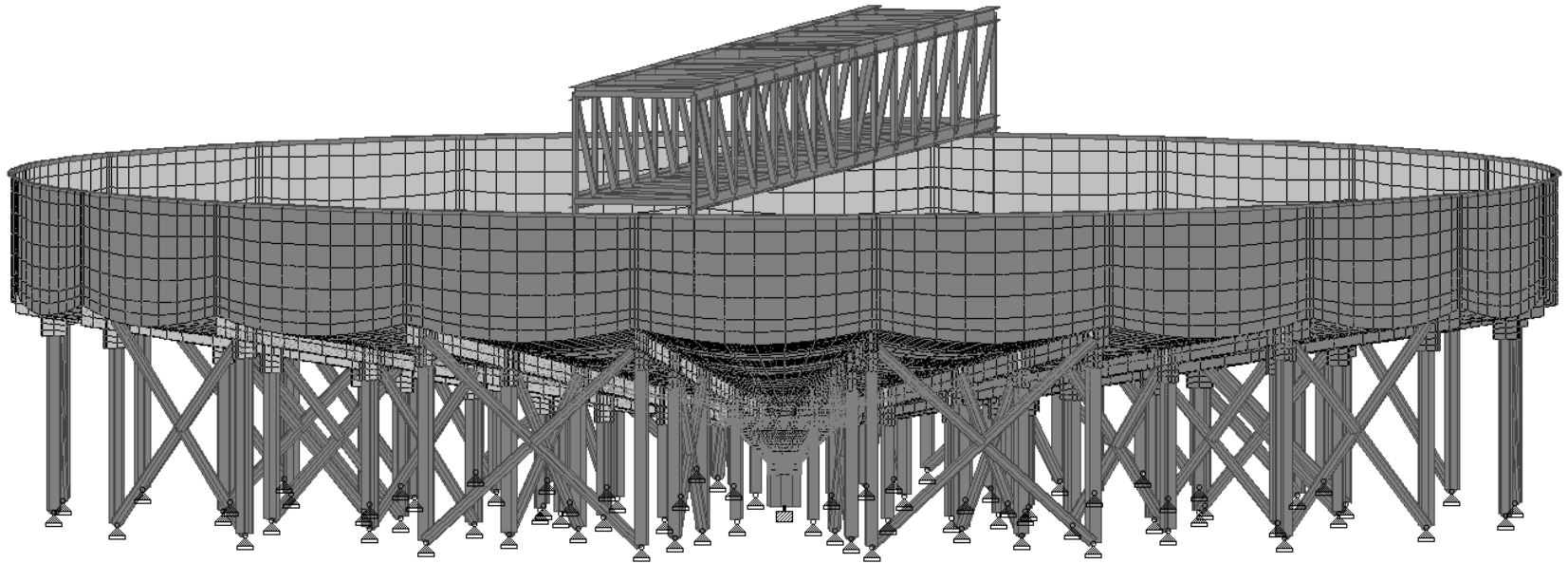
Single translational DOF spring

Single rotational DOF spring

(a) Stiffness $\mathbf{K} = [k]$

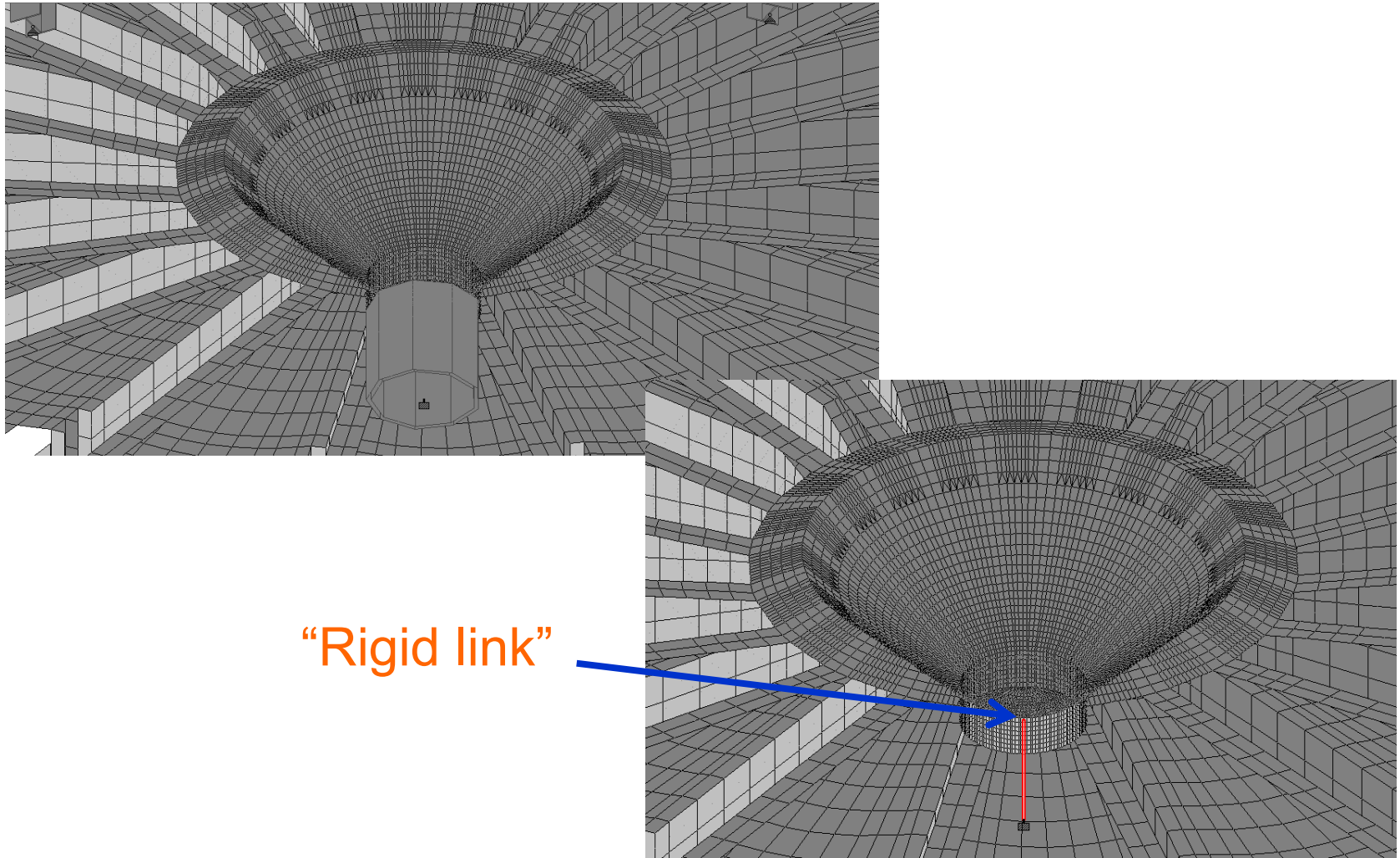
Theory and Modeling Guide,
Volume I: ADINA, Report ARD 11-8,
December 2011, Capítulo 2.9.2

- “Rigid links”




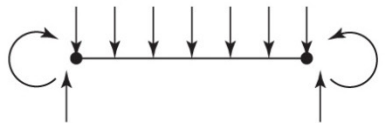
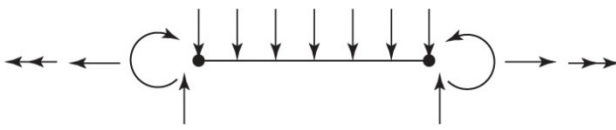
Theory and Modeling Guide, Volume I: ADINA, Report ARD
11-8, December 2011, Capítulo 5.15.2

Condiciones de Soporte



"Rigid link"

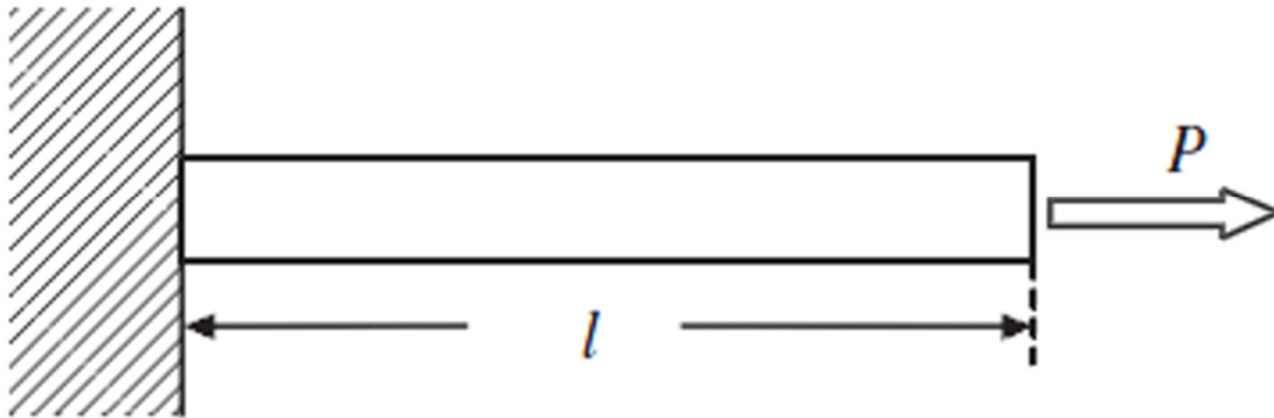
Elementos Basados en Líneas

Element Type	None	Shape	Number of Nodes	Applications
Line	Truss		2	Pin-ended bar in tension or compression
	Beam		2	Bending
	Frame		2	Axial, torsional, and bending. With or without load stiffening.

(continued)

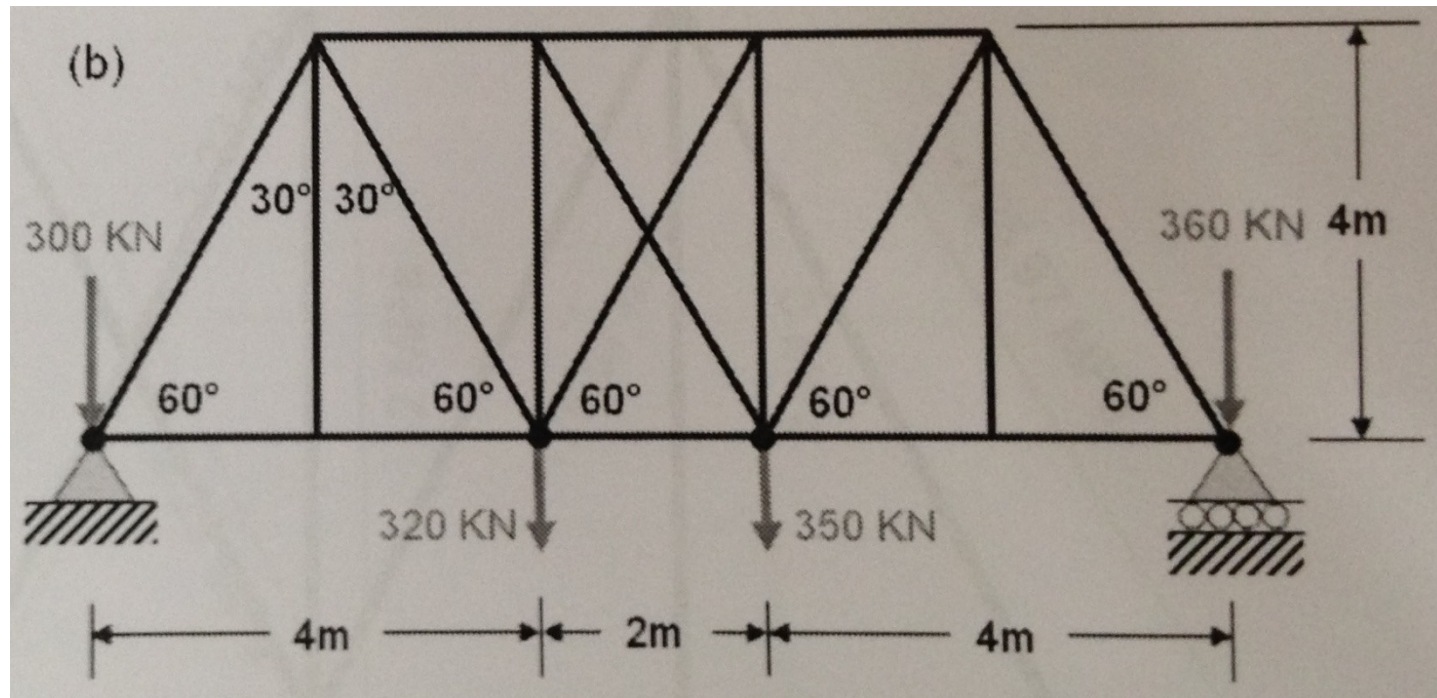
La barra solamente puede transmitir fuerzas aplicadas en los nodos

Elementos Basados en Líneas



¿Barra, viga o marco?

Elementos Basados en Líneas



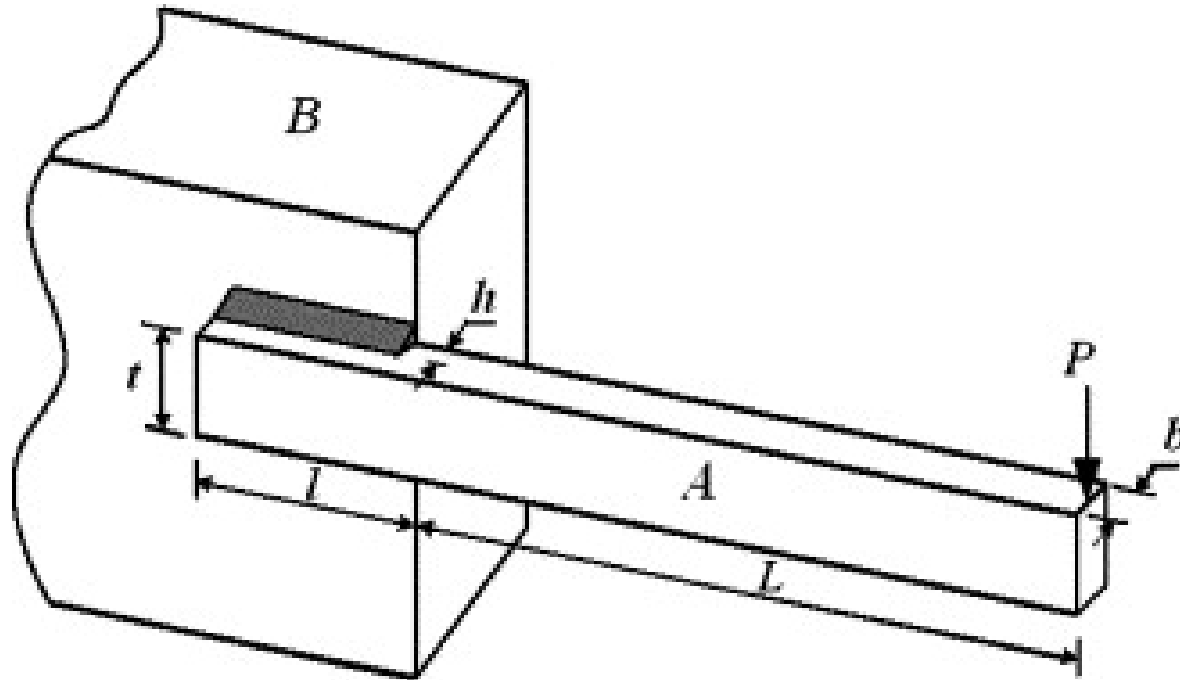
¿Barra, viga o marco?

Elementos Basados en Líneas



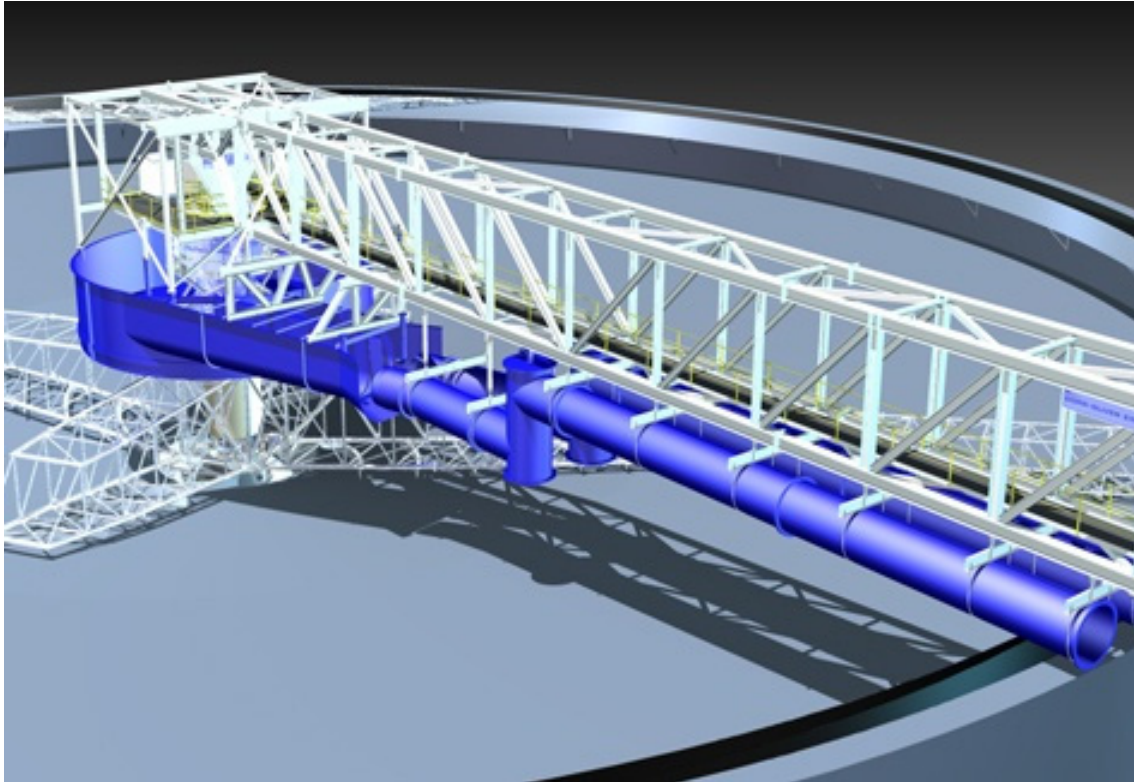
¿Barra, viga o marco? Dependerá de cómo se aplican las cargas en la estructura (nodales, distribuidas, peso propio, etc.) y de la geometría del enrejado.

Elementos Basados en Líneas



¿Barra, viga o marco?

Elementos Basados en Líneas



¿Barra, viga o marco?

Elementos Basados en Líneas



¿Barra, viga o marco?

Elementos Basados en Líneas



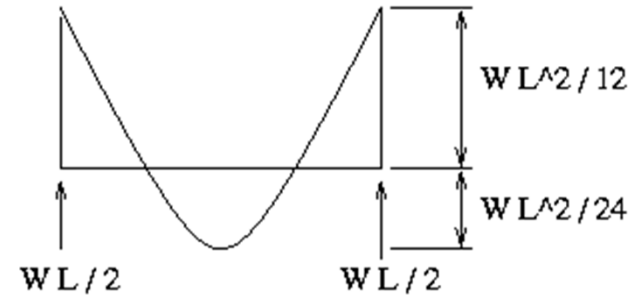
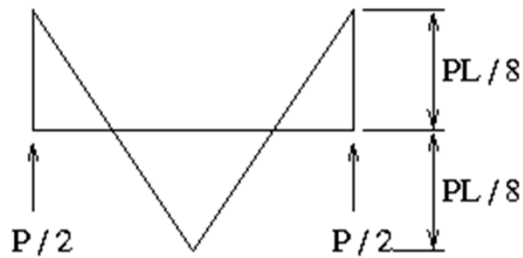
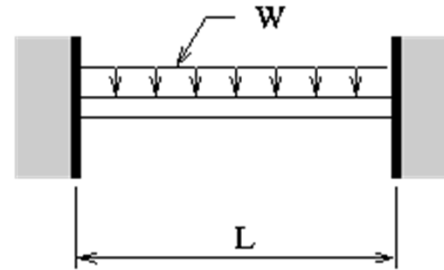
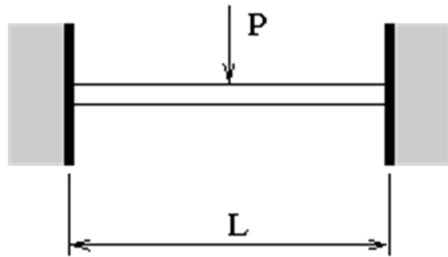
¿Barra, viga o marco?

Elementos Basados en Líneas



¿Barra, viga o marco?

Elementos Basados en Líneas



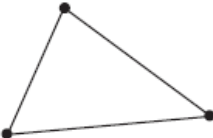



POINT LOAD IN CENTER OF BEAM

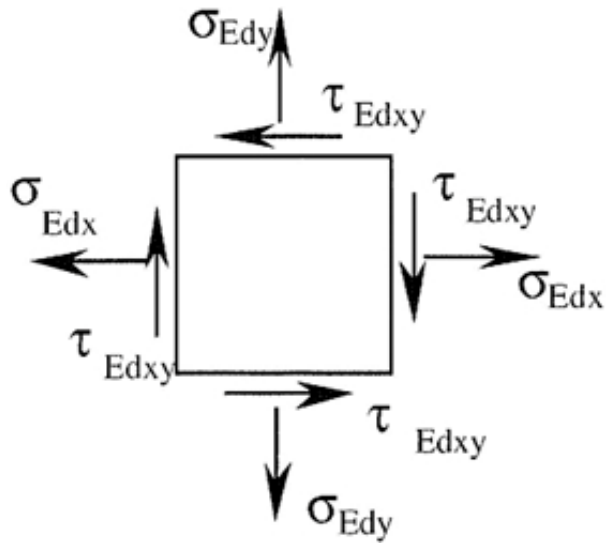
UNIFORM LOAD ALONG BEAM

¿Barra, viga o marco?

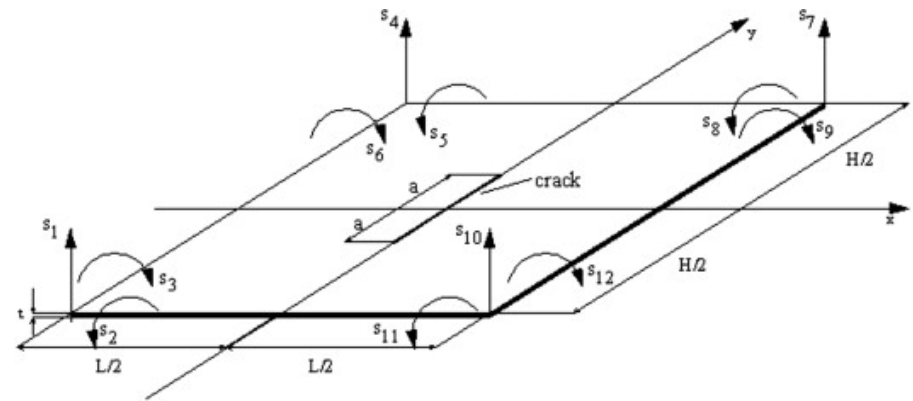
Elementos Basados en Superficies

Element Type	None	Shape	Number of Nodes	Applications
Surface	4-node quadrilateral		4	Plane stress or strain, axisymmetry, shear panel, thin flat plate in bending
	8-node quadrilateral		8	Plane stress or strain, thin plate or shell in bending
	3-node triangular		3	Plane stress or strain, axisymmetry, shear panel, thin flat plate in bending. Prefer quad where possible. Used for transitions of quads.
	6-node triangular		6	Plane stress or strain, axisymmetry, thin plate or shell in bending. Prefer quad where possible. Used for transitions of quads.

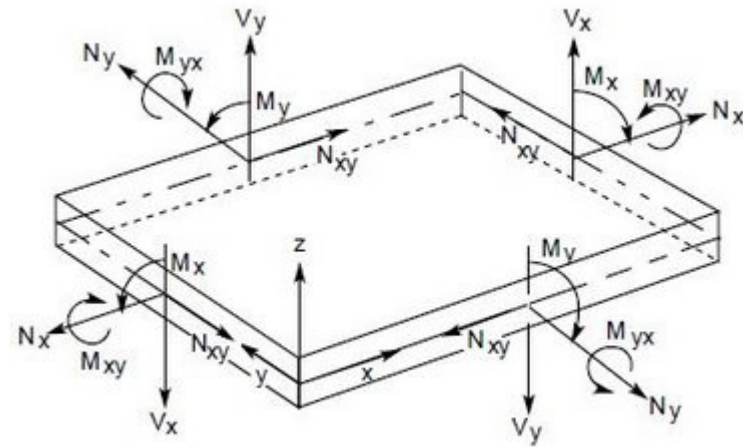
Elementos Basados en Superficies



membrana/esfuerzo
plano/deformación plana

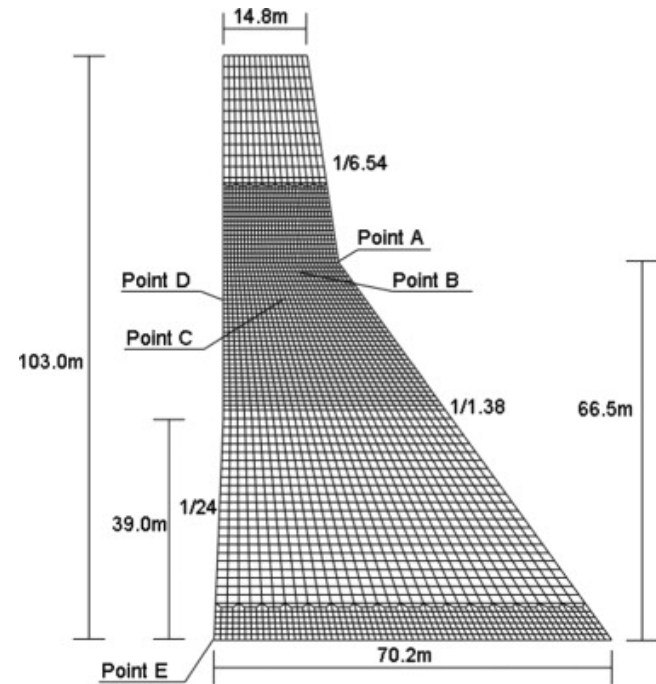


Placa



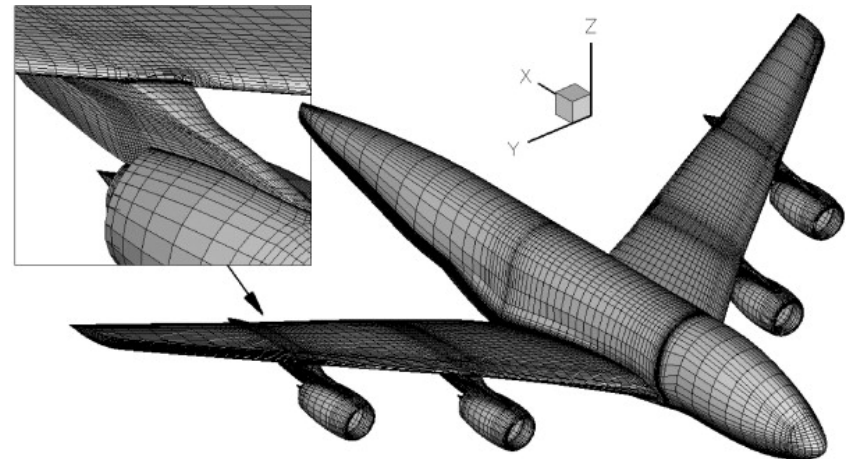
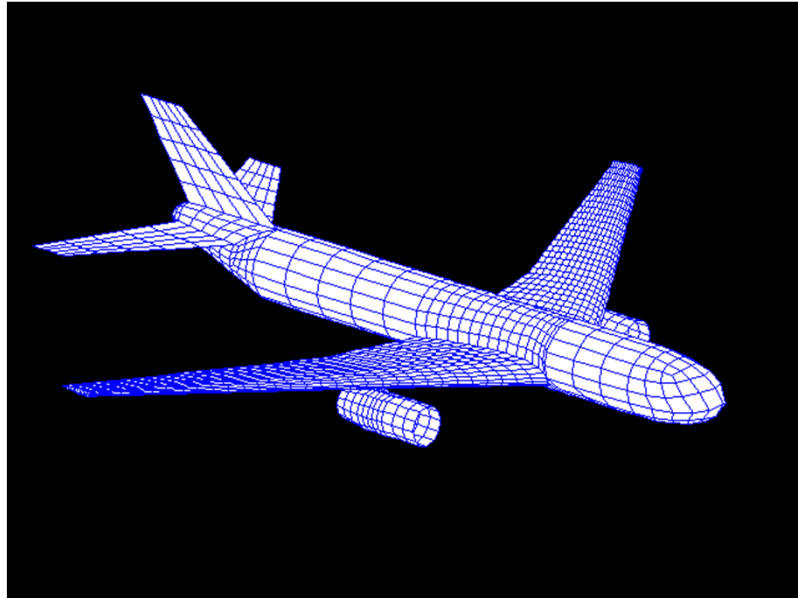
Coraza (shell)

Elementos Basados en Superficies



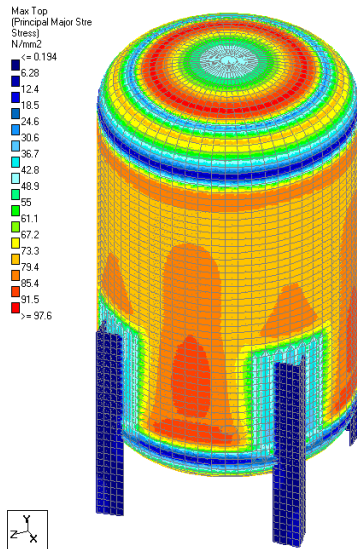
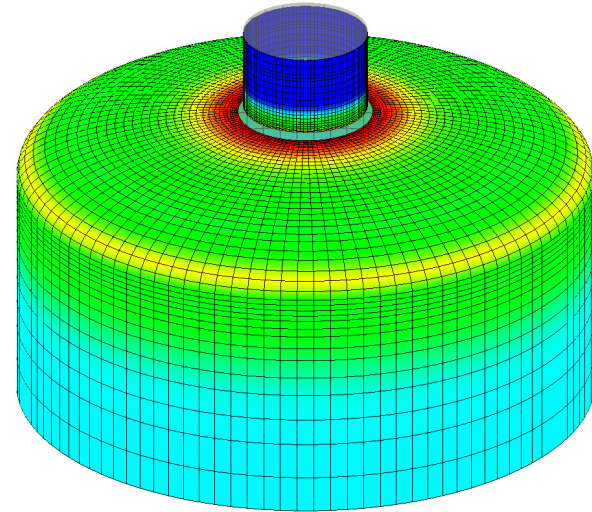
¿Esf. plano, def. plana, membrana,
axisimétrico, placa o coraza?

Elementos Basados en Superficies



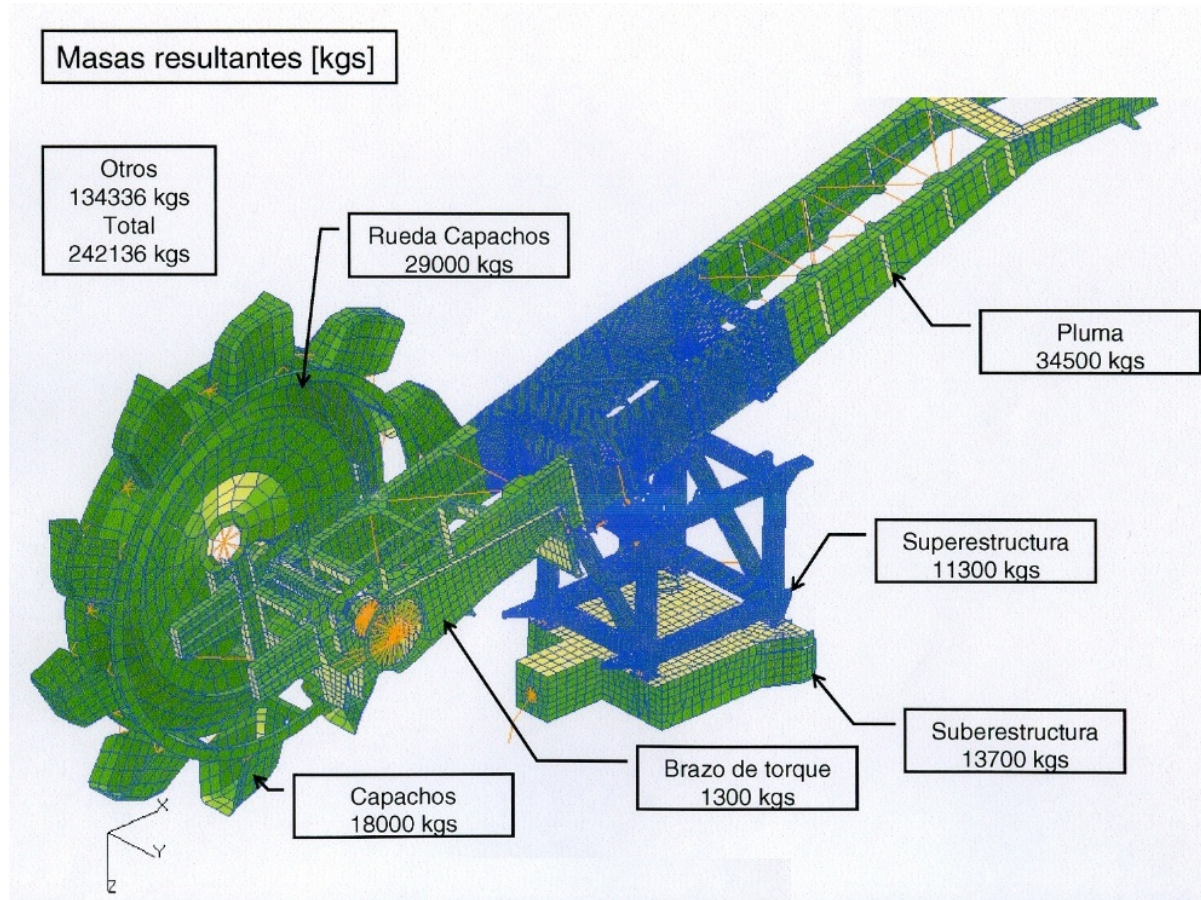
¿Esf. plano, def. plana, membrana,
axisimétrico, placa o coraza?

Elementos Basados en Superficies



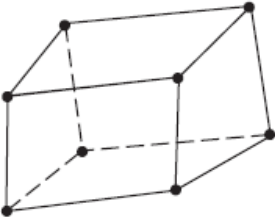
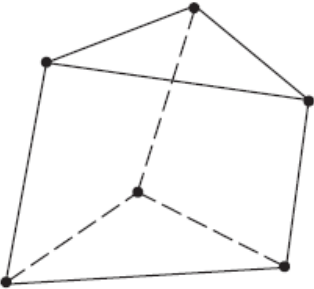
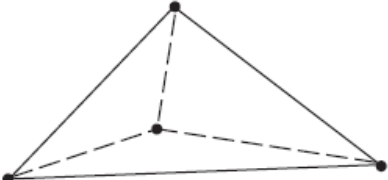
¿Esf. plano, def. plana, membrana, axisimétrico, placa o coraza?

Elementos Basados en Superficies

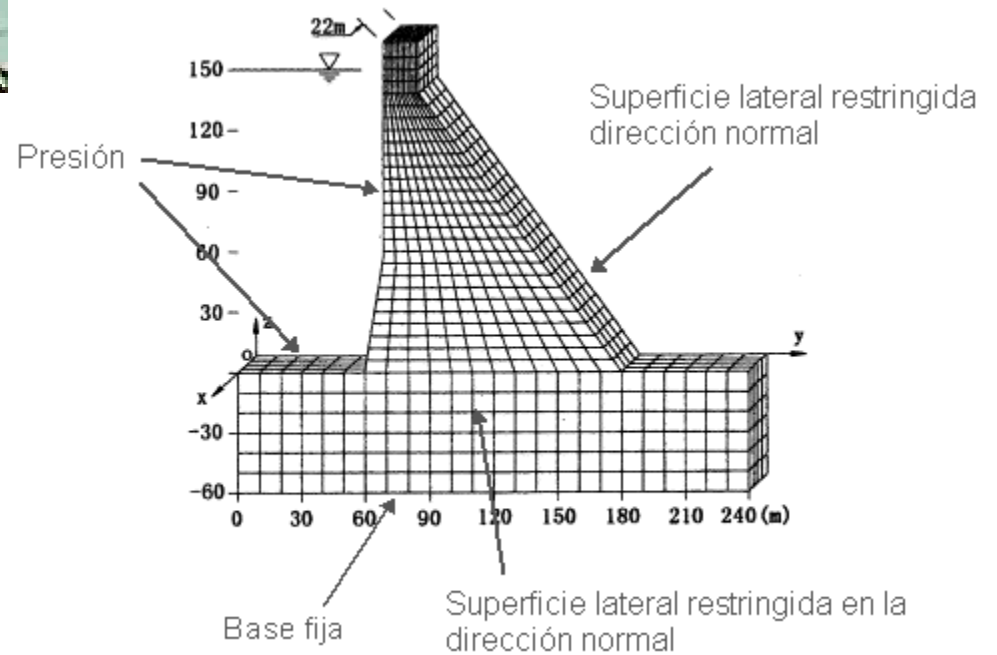


¿Esf. plano, def. plana, membrana,
axisimétrico, placa o coraza?

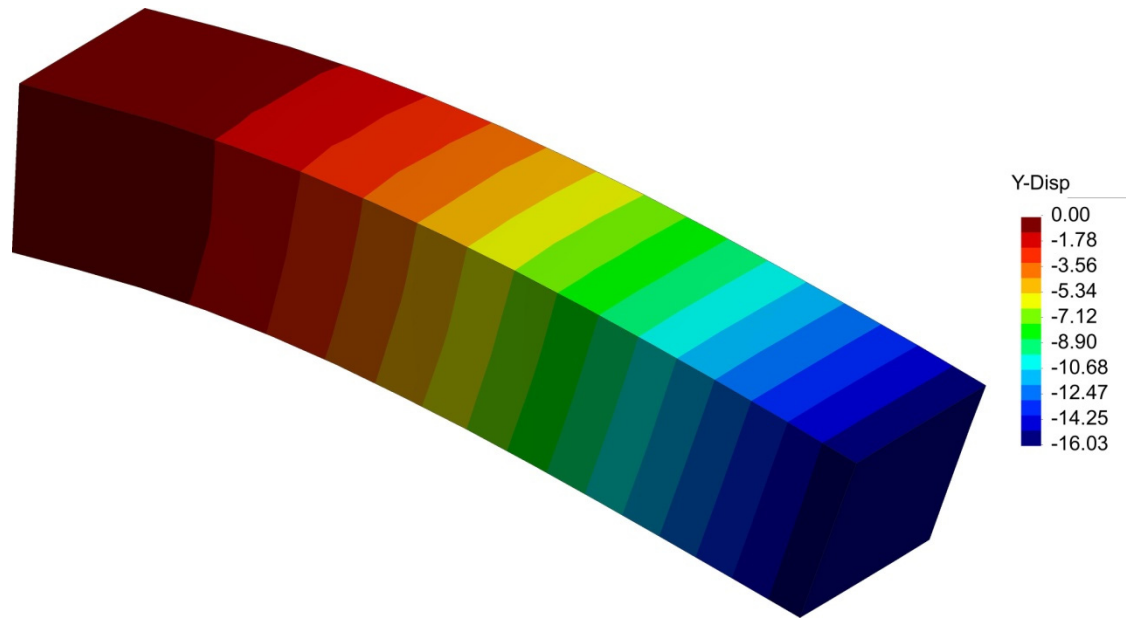
Elementos Basados en Sólidos

	8-node hexagonal (brick)		8	Solid, thick plate
Solid [†]	6-node pentagonal (wedge)		6	Solid, thick plate. Used for transitions.
	4-node tetrahedron (tet)		4	Solid, thick plate. Used for transitions.

Elementos Basados en Sólidos

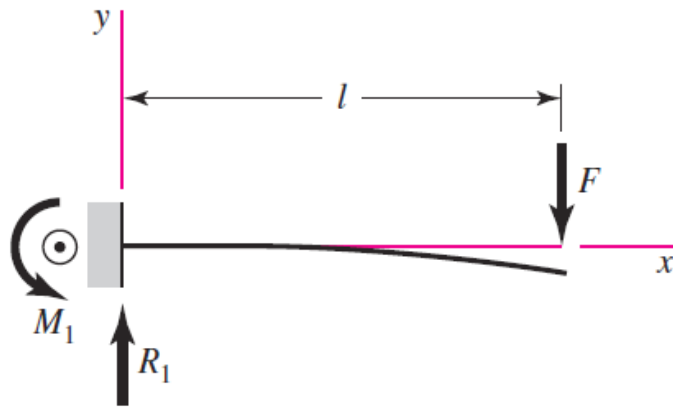


- Todos los problemas son esencialmente tridimensionales



- Muchas veces es posible simplificar el problema

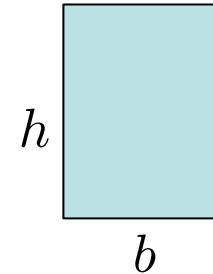
Viga de Euler-Bernoulli



$$\sigma = \frac{My}{I}$$
$$y_{\text{máx}} = -\frac{Fl^3}{3EI}$$

$$M_1 = Fl$$

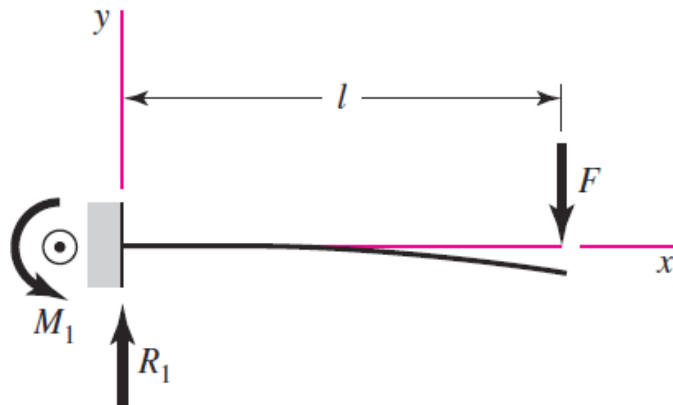
$$R_1 = F$$



$$I = \frac{bd^3}{12}$$

- Desprecia los esfuerzos de corte
- Válida cuando $y_{\text{máx}}/l \ll 1$ y $h/l \ll 1$
- Por ejemplo, $l/y_{\text{máx}} \geq 1000$ y $l/h \geq 20$

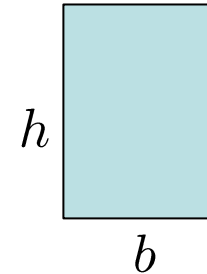
Viga de Euler-Bernoulli



$$\sigma = \frac{My}{I}$$
$$y_{\text{máx}} = -\frac{Fl^3}{3EI}$$

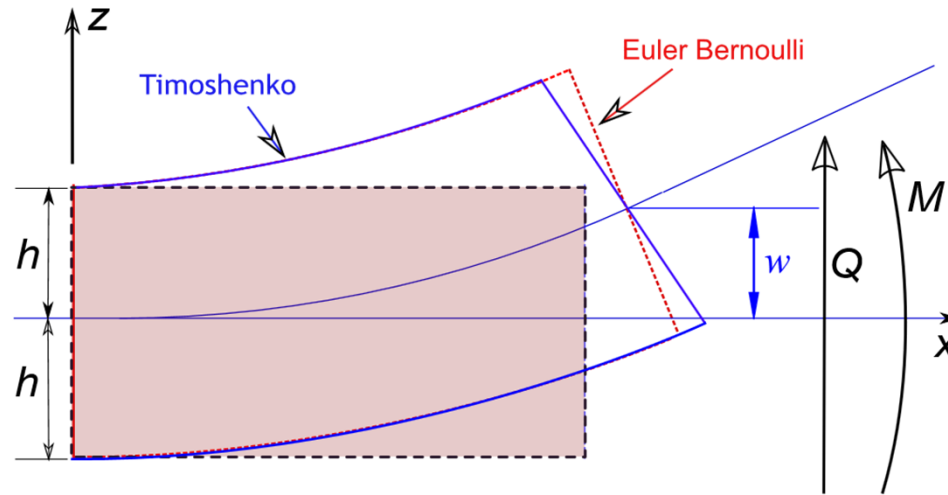
$$M_1 = Fl$$

$$R_1 = F$$



$$I = \frac{bd^3}{12}$$

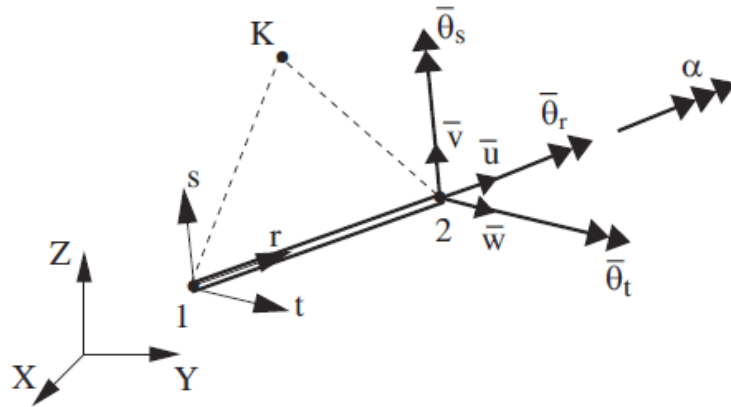
- cuando $l/h \rightarrow 1$ resultados **no coincidirán** con la realidad física
- cuando $l/h \rightarrow 1$ los esfuerzos de corte **no son despreciables**
- Teoría de Timoshenko es más apropiada



- Teoría de Timoshenko **no captura efectos tridimensionales** cuando la viga es muy corta
- Esfuerzos en el lado empotrado son tridimensionales (modelo tridimensional requerido)

- Sirve como “beam” y “frame”
- Teoría de Euler-Bernoulli (E-B)
- Teoría de Timoshenko: E-B se corrige mediante factores de área de corte (“shear area factors”)

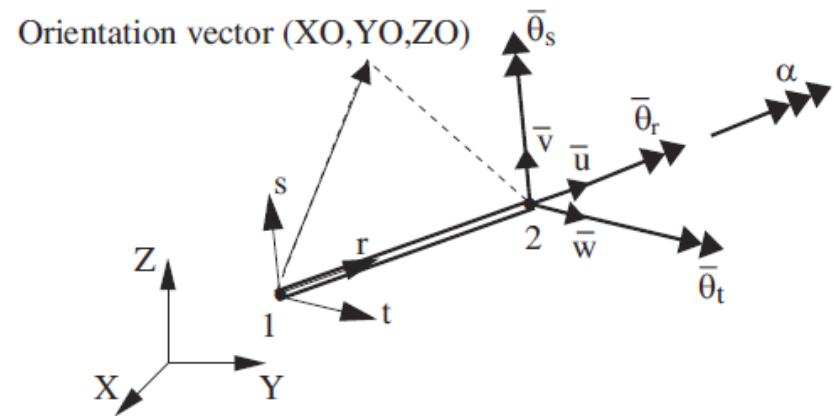
Definición de la orientación



The s-direction lies in the plane defined by nodes 1, 2, K, and the t-direction is perpendicular to the r-s plane.

α : warping DOF

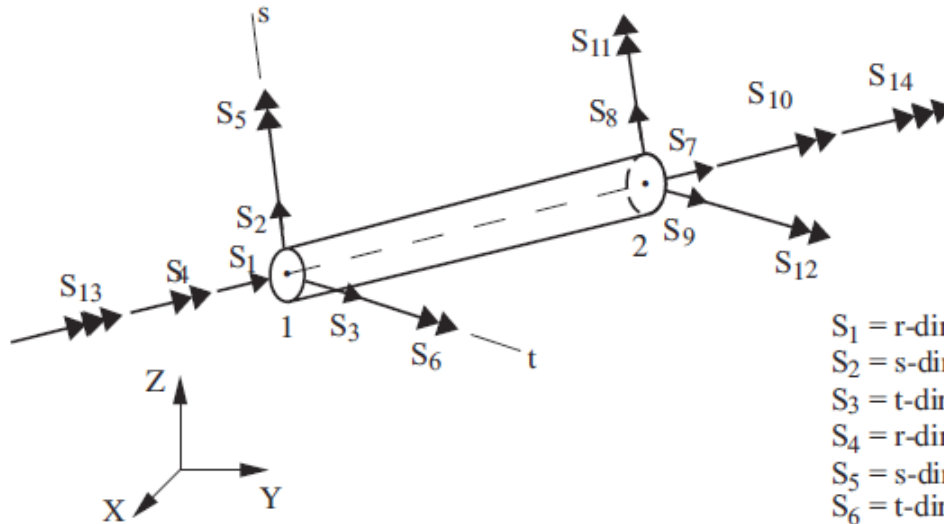
(a) Geometry definition, using auxiliary node K



The s-direction lies in the plane defined by nodes 1, 2 and the orientation vector. The t-direction is perpendicular to the r-s plane.

(b) Geometry definition, using orientation vector

Variables de salida (fuerzas y momentos)



S_1 = r-direction force at node 1 (axial force, positive in compression)

S_2 = s-direction force at node 1 (shear force)

S_3 = t-direction force at node 1 (shear force)

S_4 = r-direction moment at node 1 (torsion)

S_5 = s-direction moment at node 1 (bending moment)

S_6 = t-direction moment at node 1 (bending moment)

S_7 = r-direction force at node 2 (axial force, positive in tension)

S_8 = s-direction force at node 2 (shear force)

S_9 = t-direction force at node 2 (shear force)

S_{10} = r-direction moment at node 2 (torsion)

S_{11} = s-direction moment at node 2 (bending moment)

S_{12} = t-direction moment at node 2 (bending moment)

S_{13} = bimoment at node 1

S_{14} = bimoment at node 2

Figure 2.4-2: Element end forces/moments

Relajación de grados de libertad (ADINA: “beam release”)

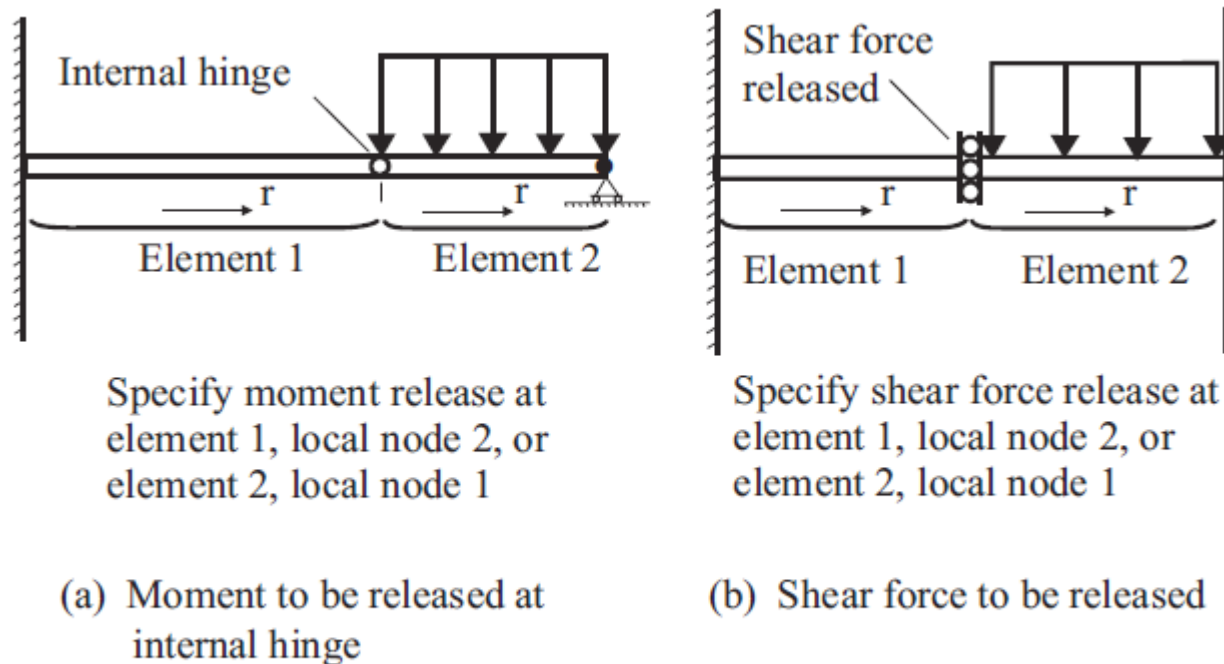
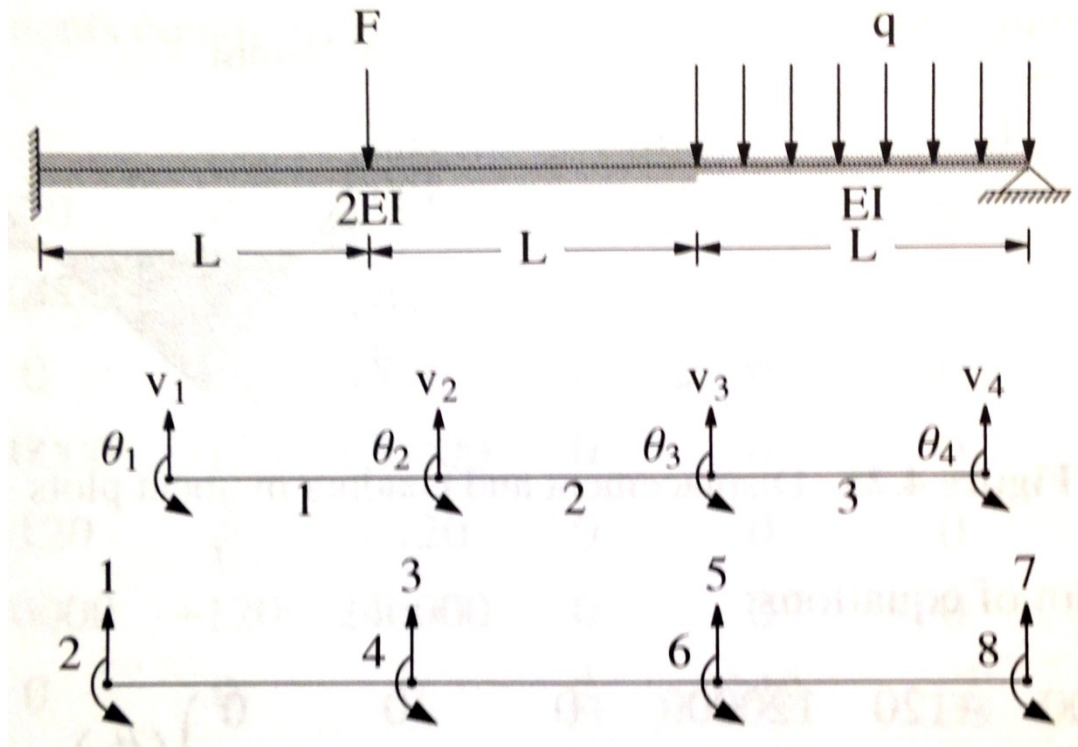


Figure 2.4-26: Use of moment and shear force release options

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December 2011, Capítulo 2.4.4

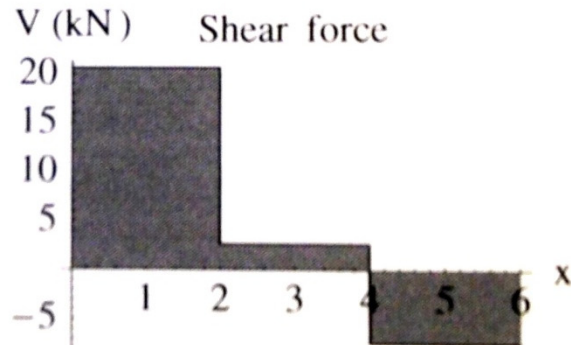
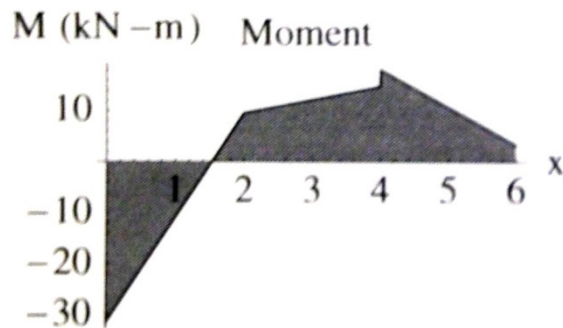
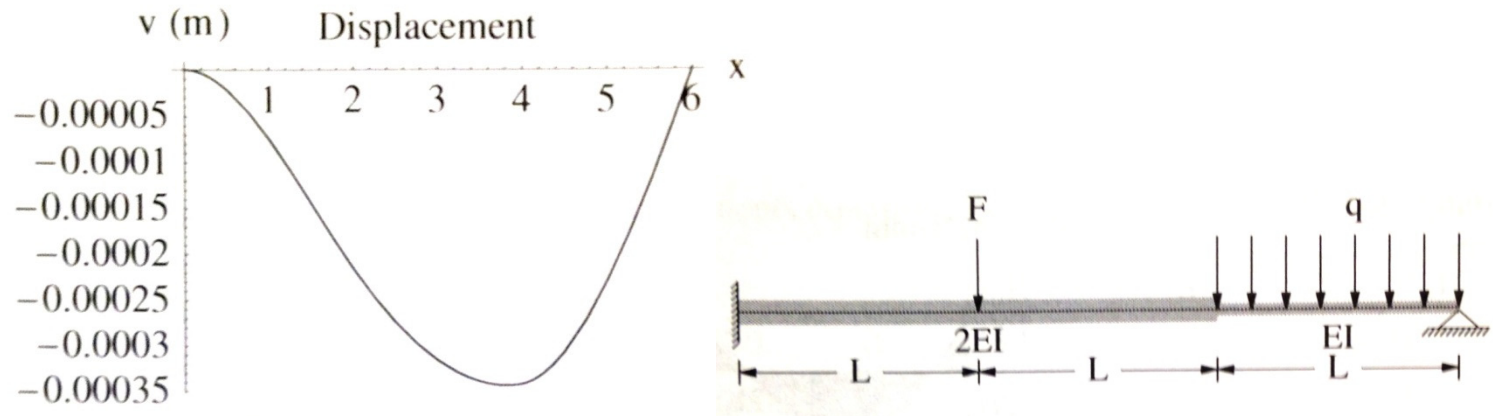
Exactitud y Convergencia del Elemento Viga



$$L = 2 \text{ m}; \quad F = 18 \text{ kN}; \quad q = 10 \text{ kN/m};$$

$$E = 210 \text{ GPa}; \quad I = 4 \times 10^{-4} \text{ m}^4$$

Exactitud y Convergencia del Elemento Viga



Solución con 1 elemento por tramo

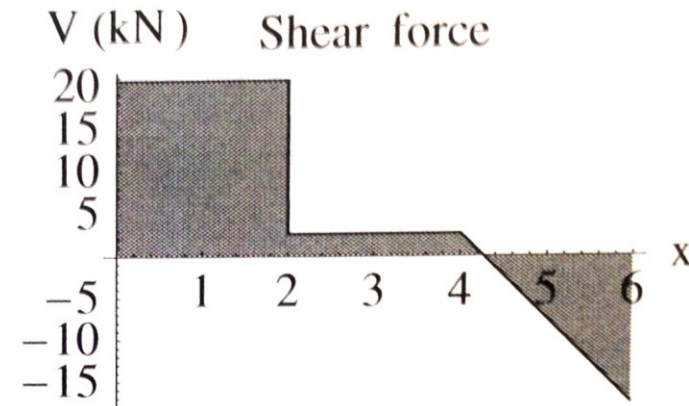
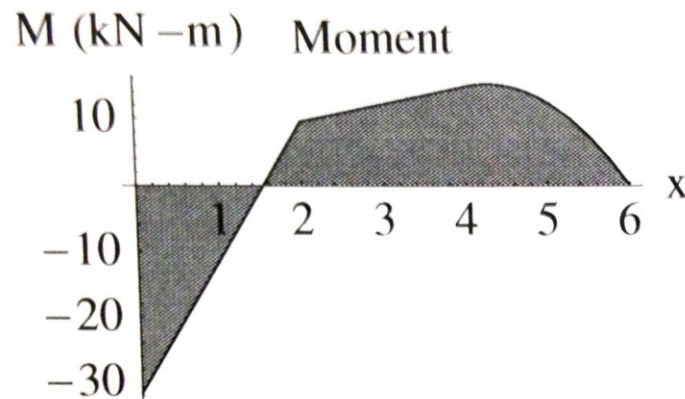
Exactitud y Convergencia del Elemento Viga

Solución analítica tercer tramo

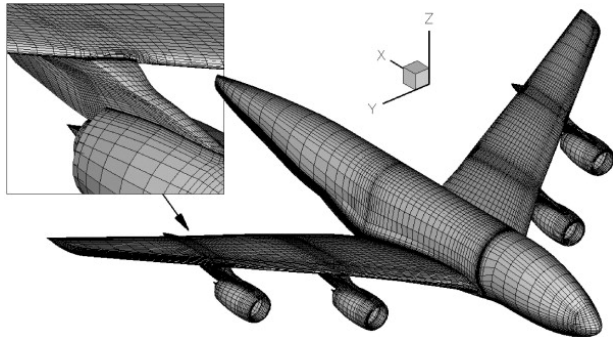
$$M(x) = EI d^2 v/dx^2 = -5.x^2 + 42.6071x - 75.6429$$

$$V(x) = EI d^3 v/dx^3 = 42.6071 - 10.x$$

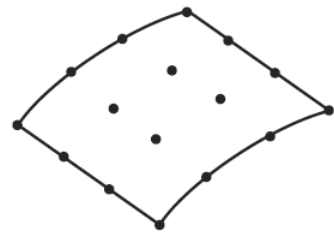
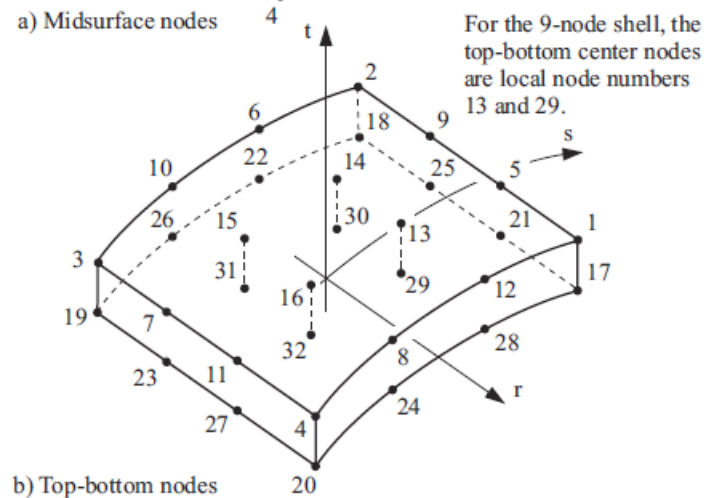
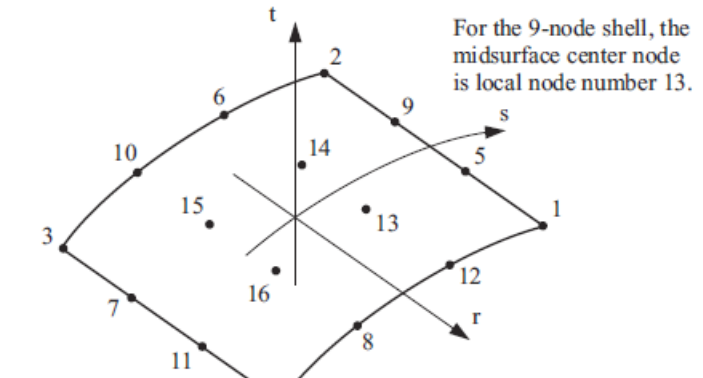
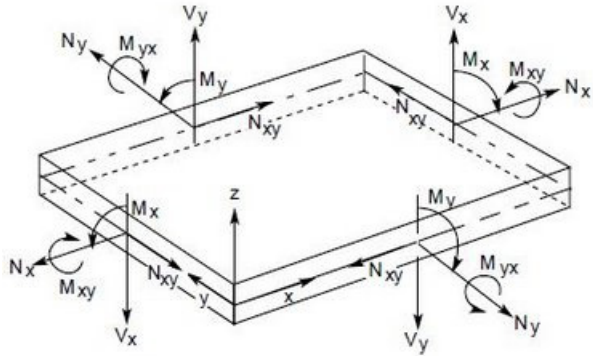
Solución con 1 elemento en el primer y segundo tramo, y 5 elementos en el tercer tramo



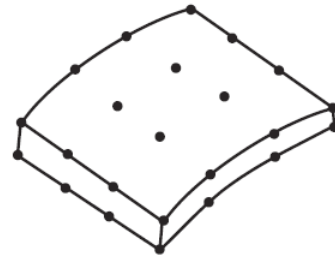
ADINA: Elementos “Plate” / “Shell”



ADINA: Elementos "Plate" / "Shell"



a) Midsurface description



c) Top-bottom description

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