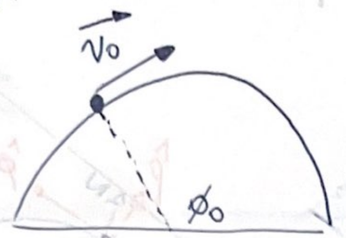
Aux 4

(1)

1) Tenemos: Donde $\vec{v_0} = v_0 \hat{\phi}_0$



y queremos encontrar la posición en que la partícula se despega:

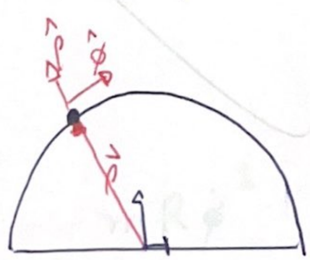
Para ello:

1) Fijar sistema de coordenadas

2) Ver las Fuerzas del problema $(\vec{F} = m\vec{a})$

3) Cond. de despegue.

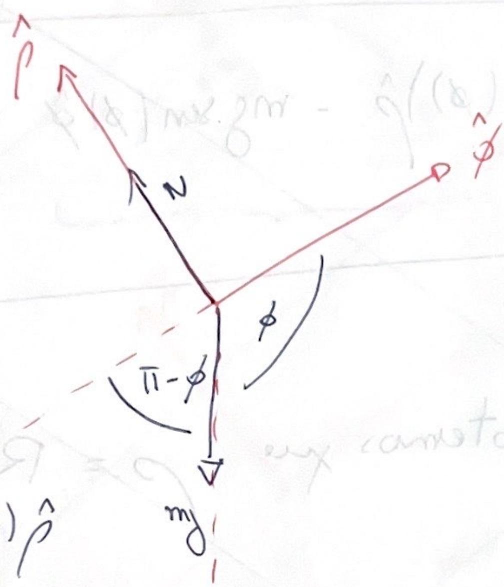
1) Fijamos sistema de coordenadas polares en el centro de la semicircunferencia



2) Anotemos la aceleración en polares:

$$\vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi}$$

y las fuerzas:



→ Tenemos que:

$$F = N\hat{\rho} - mg\,\mathrm{sen}(\pi - \phi)\hat{\rho} - mg\cos(\pi - \phi)\hat{\phi}$$

Donde usando que $\mathrm{sen}(\pi - x) = \mathrm{sen}(x)$

$\cos(\pi - x) = -\cos(x)$

$F = (N - mg\,sen(\phi))\hat{\rho} + mg\cos\phi\,\hat{\phi}$

Luego usamos $\vec{F} = m\vec{a}$

$$(N - mg\,sen\,\phi)\hat{\rho} + mg\cos\phi\,\hat{\phi} = m(\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + m(2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi}$$

Notemos que $\rho = R \rightarrow \dot{\rho} = \ddot{\rho} = 0$

$$(N - mg\,sen\phi)\hat{\rho} + mg\cos\phi\,\hat{\phi} = -mR\dot{\phi}^2\hat{\rho} + mR\ddot{\phi}\,\hat{\phi}$$

Separando en ejes:

$\hat{\rho})$ $N - mg\,sen\phi = -mR\dot{\phi}^2$ $*$

$\hat{\phi})$ $mg\cos\phi = mR\ddot{\phi}$ $**$

Viendo a ✩✩ y con ello despejamos $\dot{\phi}$:

$$\cancel{m} g \cos\phi = \cancel{m} R \ddot{\phi}$$

$$\frac{g}{R}\cos\phi = \ddot{\phi}$$

$$\frac{g}{R}\cos\phi = \frac{d\dot{\phi}}{d\phi}\frac{d\phi}{dt}$$

$$\frac{g}{R}\cos\phi = \frac{d\dot{\phi}}{d\phi}\dot{\phi}$$

$$\int_{\phi_0}^{\phi} d\phi$$

$$\int_{\phi_0}^{\phi}\frac{g}{R}\cos\phi = \int_{\dot{\phi}_0}^{\dot{\phi}}\frac{d\dot{\phi}}{\cancel{d\phi}}\dot{\phi}\, \cancel{d\phi}$$

$$\frac{g}{R}(\operatorname{sen}\phi - \operatorname{sen}\phi_0) = \dot{\phi}^2/2 - \frac{\dot{\phi}_0^2}{2}$$

la velocidad angular inicial $\dot{\phi}_0$ se puede obtener de que la velocidad inicial es $\vec{v}_0 = v_0\hat{\phi}$

$= R\dot{\phi}_0\hat{\phi}$

$$\dot{\phi}_0 = \frac{v_0}{R}$$

$$\frac{g}{R}(\operatorname{sen}\phi - \operatorname{sen}\phi_0) = \frac{\dot{\phi}^2}{2} - \frac{v_0^2}{2R^2}$$

$$\boxed{2\left(\frac{g}{R}(\operatorname{sen}\phi - \operatorname{sen}\phi_0) + \frac{v_0^2}{2R^2}\right) = \dot{\phi}^2}$$ ✩✩✩

Usando ☆☆☆ en ☆

$$N - mg\,\text{sen}\phi = -mR\left(2\left(\frac{g}{R}(\text{sen}\phi - \text{sen}\phi_0) + \frac{v_0^2}{2R^2}\right)\right)$$

y se despega cuando $N=0$

$$\cancel{m}g\,\text{sen}\phi = \cancel{m}R\left(2\left(\frac{g}{R}(\text{sen}\phi - \text{sen}\phi_0)\right) + \frac{v_0^2}{R^2}\right)$$

$$\cancel{m}g\,\text{sen}\phi = 2g\,\text{sen}\phi - 2g\,\text{sen}\phi_0 + \frac{v_0^2}{R}$$

$$g\,\text{sen}\phi = 2g\,\text{sen}\phi_0 - \frac{v_0^2}{R}$$

$$\text{sen}\phi = 2\,\text{sen}\phi_0 - \frac{v_0^2}{gR}$$

$$\boxed{\phi_{despegue} = \arcsin\left(2\,\text{sen}\phi_0 - \frac{v_0^2}{gR}\right)}$$

P2)

La trayectoria esta dada por

$$\rho = A e^{k\theta} \quad , \quad z = h\rho$$

con $|\vec{v}| = v_0 = cte$:

a) queremos encontrar $\vec{v}$:

Sabemos que en cilíndricas:

$$\vec{r} = \rho\hat{\rho} + z\hat{z}$$

$$\dot{\vec{r}} = \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$$

$$\ddot{\vec{r}} = \vec{a} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}$$

$$\rightarrow \quad \vec{v} = \dot{\rho}\hat{\rho} + \rho\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$$

(($\phi = \theta$) cambie de notación, pero $\hat{\theta} = \hat{\phi}$)

y sabemos que $\rho = Ae^{\kappa\theta}$, $z = h\rho$

$$\rightarrow \dot{\rho} = A\kappa e^{\kappa\theta}\dot{\theta}$$

$$\dot{z} = h\dot{\rho} = hA\kappa e^{\kappa\theta}\dot{\theta}$$

Luego:

$$\boxed{\vec{v} = A\kappa e^{\kappa\theta}\dot{\theta}\hat{\rho} + Ae^{\kappa\theta}\dot{\theta}\hat{\theta} + hA\kappa e^{\kappa\theta}\dot{\theta}\hat{z}}$$

como sabemos que $|\vec{v}| = v_0$

$$|\vec{v}| = \sqrt{(A\kappa e^{\kappa\theta}\dot{\theta})^2 + (Ae^{\kappa\theta}\dot{\theta})^2 + (hA\kappa e^{\kappa\theta}\dot{\theta})^2} = v_0$$

$$= Ae^{\kappa\theta}\dot{\theta}\sqrt{\kappa^2 + 1 + h^2\kappa^2} = v_0$$

$$\Rightarrow Ae^{\kappa\theta}\dot{\theta} = \frac{v_0}{\sqrt{\kappa^2 + 1 + h^2\kappa^2}}$$

Luego usando esto en ✱

$$\boxed{\vec{v} = \frac{v_0}{\sqrt{\kappa^2+1+h^2\kappa^2}} \left(\kappa\hat{\rho} + \hat{\theta} + h\kappa\hat{z}\right)}$$

b) Para la aceleración debemos derivar la velocidad

como $\longrightarrow$ $\frac{d}{dt}\hat{\rho} = \dot{\theta}\hat{\theta}$ $\qquad \frac{d}{dt}\hat{z} = \hat{z}$

$$\frac{d}{dt}\hat{\theta} = -\hat{\rho}\dot{\theta}$$

$$\boxed{\vec{a} = \frac{v_0}{\sqrt{\kappa^2+1+h^2\kappa^2}} \left(\kappa\dot{\theta}\hat{\theta} - \hat{\rho}\right)}$$

donde como vimos antes $Ae^{\kappa\theta}\dot{\theta} = \frac{v_0}{\sqrt{\kappa^2+1+h^2\kappa^2}}$

$$\dot{\theta} = \frac{v_0}{Ae^{\kappa\theta}\sqrt{\kappa^2+1+h^2\kappa^2}}$$

$\longrightarrow$

$$\vec{a} = \frac{v_0^2}{Ae^{k\theta}(k^2+1+n^2k^2)}\left(k\hat{\theta} \mp \hat{\rho}\right)$$

c) para que sean perpendiculares $\vec{a}\cdot\vec{v} = 0$

para ello notemos que $|\vec{v}| = v_0$

$$\vec{v}\cdot\vec{v} = v_0^2 \quad \Big/ \frac{d}{dt}$$

$$\frac{d}{dt}(\vec{v}\cdot\vec{v}) = 0$$

$$\vec{a}\cdot\vec{v} + \vec{v}\cdot\vec{a} = 0$$

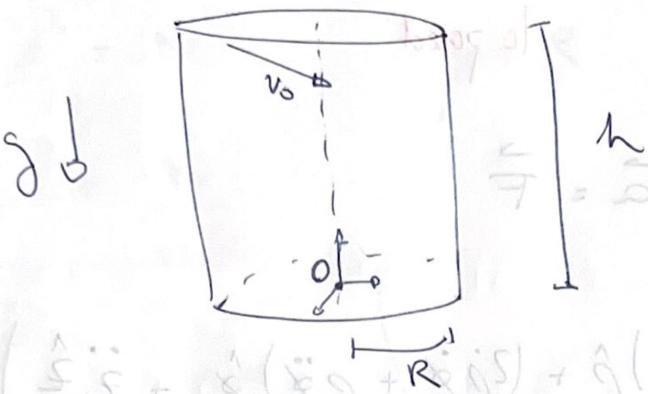
$$2\vec{a}\cdot\vec{v} = 0$$

$$\boxed{\vec{a}\cdot\vec{v} = 0}$$

$\hookrightarrow$ son perpendiculares.

P3)

Tenemos



Colocamos nuestro sistema de coordenadas en el medio de la cara inferior del cilindro:

Como es cilindro ⟶ coordenadas cilíndricas:

$$\vec{r} = \rho\hat{\rho} + z\hat{z}$$

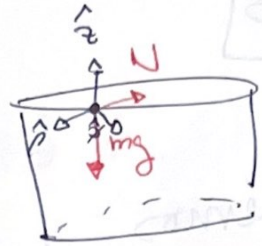
$$\vec{a} = \ddot{\vec{r}} = (\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}$$

Ya con la aceleración, hacemos $\vec{F} = m\vec{a}$

Donde $\vec{F} = -N\hat{\rho} - mg\hat{z}$

normal de la pared

peso



Usando $m\vec{a} = \vec{F}$

$$m\left((\ddot{\rho} - \rho\dot{\phi}^2)\hat{\rho} + (2\dot{\rho}\dot{\phi} + \rho\ddot{\phi})\hat{\phi} + \ddot{z}\hat{z}\right) = -N\hat{\rho} - mg\hat{z}$$

Donde $\rho = R \rightarrow \dot{\rho} = \ddot{\rho} = 0$

$$\underbrace{m\left(-\rho\dot{\phi}^2\hat{\rho} + \rho\ddot{\phi}\hat{\phi} + \ddot{z}\hat{z}\right) = -N\hat{\rho} - mg\hat{z}}_{\text{3 ecuaciones}}$$

$\hat{\rho}]$ $\quad -m\rho\dot{\phi}^2 = -N$

$\hat{\phi}]$ $\quad m\rho\ddot{\phi} = 0$

$\hat{z}]$ $\quad m\ddot{z} = -mg$

de la primera:

$$-m\rho\dot{\phi}^2 = -N$$

$$m\rho\dot{\phi}^2 = N \qquad \rho = R$$

$$\dot{\phi}^2 = \frac{N}{mR}$$

$$\boxed{\dot{\phi} = \sqrt{\frac{N}{mR}}}$$

de la 3era:

$$\cancel{m}\ddot{z} = -\cancel{m}g$$

$$\ddot{z} = -g \qquad \int_{t=0}^{t=t} dt$$

$$\dot{z}(t) - \cancel{\dot{z}(0)} = -gt$$

0 parte con vel tangencial.

$$\boxed{\dot{z}(t) = -gt} \longrightarrow \text{velocidad vertical}$$

de la segunda

$m\rho\ddot{\phi} = 0$ $\qquad \rho = R$

$mR\ddot{\phi} = 0$ $\qquad \int_{t=0}^{t=t} dt$

$mR(\dot{\phi}(t) - \dot{\phi}(t=0)) = 0$

$\dot{\phi}(t) = \dot{\phi}(t=0)$

como la velocidad inicial es tangencial $= v_0$

como el mov es circular en $\hat{\rho}, \hat{\phi}$ la velocidad angular inicial es $R\dot{\phi}(t=0) = v_0$

$\dot{\phi}(t=0) = \frac{v_0}{R}$

$$\boxed{\dot{\phi}(t) = \frac{v_0}{R}}$$ velocidad angular.

Resumen

Vector posición : $\vec{r}$

" velocidad : $\vec{v} = \dot{\vec{r}}$

" aceleración : $\vec{a} = \dot{\vec{v}} = \ddot{\vec{r}}$

Coordenadas

1) Cartesianas :

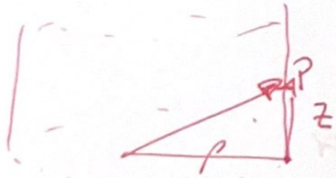
$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$\dot{\vec{r}} = \dot{x}\,\hat{x} + \dot{y}\,\hat{y} + \dot{z}\,\hat{z}$$

$$\ddot{\vec{r}} = \ddot{x}\,\hat{x} + \ddot{y}\,\hat{y} + \ddot{z}\,\hat{z}$$

2) Cilíndricas :

$$\vec{r} = \rho\,\hat{\rho} + z\,\hat{z}$$



$$\dot{\vec{r}} = \dot{\rho}\,\hat{\rho} + \rho\dot{\theta}\,\hat{\theta} + \dot{z}\,\hat{z}$$

$$\ddot{\vec{r}} = (\ddot{\rho} - \rho\dot{\theta}^2)\,\hat{\rho} + (2\dot{\rho}\dot{\theta} + \rho\ddot{\theta})\,\hat{\theta} + \ddot{z}\,\hat{z}$$

esféricos:

$$\vec{r} = r\,\hat{r}$$

$$\dot{\vec{r}} = \dot{r}\,\hat{r} + r\dot{\theta}\,\hat{\theta} + r\dot{\phi}\,\mathrm{sen}\,\theta\,\hat{\phi}$$

$$\ddot{\vec{r}} = (\ddot{r} - r\dot{\phi}^2\,\mathrm{sen}^2\theta - r\dot{\theta}^2)\,\hat{r}$$

$$+$$

$$(r\ddot{\theta} - r\dot{\phi}^2\,\mathrm{sen}\theta\cos\theta + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$+$$

$$(2\dot{r}\dot{\phi}\,\mathrm{sen}\theta + r\ddot{\phi}\,\mathrm{sen}\theta + 2r\dot{\theta}\dot{\phi}\cos\theta)\,\hat{\phi}$$

Trucos clásicos

1) $\ddot{x} = \text{algo} \qquad / \cdot \dot{x}$

$$\underbrace{\ddot{x}\cdot\dot{x}} = \text{algo} \cdot \dot{x}$$

$$\frac{d}{dt}\left(\frac{\dot{x}^2}{2}\right) =$$

2) $\ddot{x} = \dfrac{d\dot{x}}{dt} = \dfrac{d\dot{x}}{dx}\dfrac{dx}{dt}$, luego integrar.