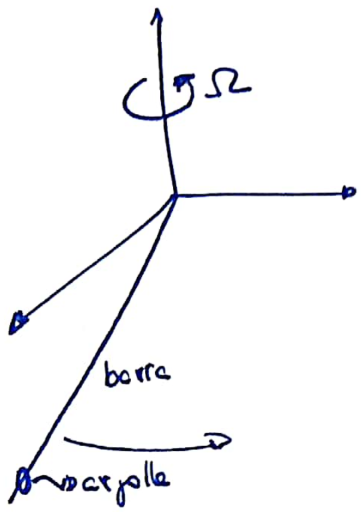
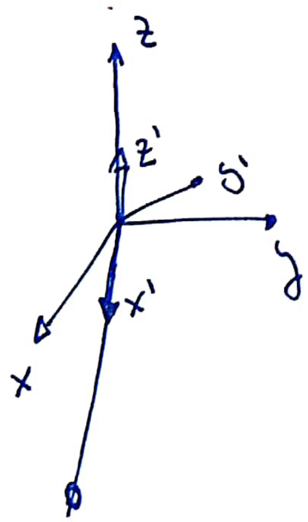
Pauta Aux Pre control 2

P1

El problema se visualiza así:



1) primero debemos elegir un sistema S': elegiremos uno que gira con Ω y que ve a la masa (argolla) solo acercarse o alejarse:



Ahora queremos escribir las ec. de mov. para S'

$$m\vec{a}' = \vec{F} + \vec{F}_{NI}$$

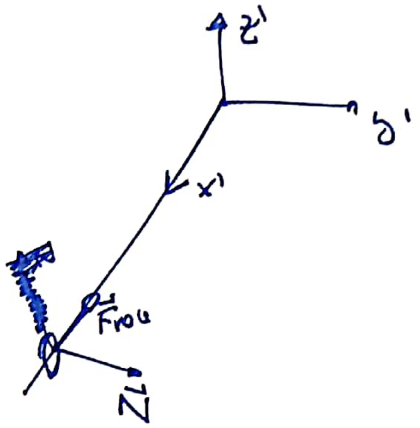
$$m\vec{a}' = m\ddot{\rho}\,\hat{x}' = \underbrace{\vec{F}}_{\vec{F}_{roce}+\vec{N}} + \underbrace{\vec{F}_{NS}}_{-2m(\vec{\Omega}\times\vec{v}') + -m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}')}$$

Solo aportan estas 2

$\vec{A}_O = 0$, no se alejan los S, $\dot{\Omega} = 0 \Rightarrow \dot{\vec{\Omega}} \times r' = 0$

$$\vec{F}_{roce} = -\mu N \hat{x}'$$

$$\vec{N} = N\hat{y}'$$



$$-2m(\vec{\Omega}\times\vec{v}') = -2m(\Omega\hat{z}' \times \dot{\rho}\hat{x}')$$

$$= -2m\Omega\dot{\rho}\,(\underbrace{\hat{z}'\times\hat{x}'}_{\hat{y}'})$$

$$= -2m\Omega\dot{\rho}\,\hat{y}'$$

$$-m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') = -m\vec{\Omega}\times(\Omega\hat{z}'\times\rho\hat{x}')$$

$$= -m\vec{\Omega}\times(\rho\Omega\hat{y}')$$

$$= -m\Omega\rho\Omega(\underbrace{\hat{z}'\times\hat{y}'}_{-\hat{x}'})$$

$$= m\Omega^2\rho\,\hat{x}'$$

→ queda

en $\hat{x}$) $\quad m\ddot{\rho} = -\mu N + m\Omega^2\rho$ ①

$\hat{y}$) $\quad 0 = N - 2m\Omega\dot{\rho}$ ②

y queremos $N(\rho)$ → de ② $\boxed{N = 2m\Omega\dot{\rho}}$ ✱

3) luego nos piden que no se mueve → $\dot{\rho} = 0$ y $\ddot{\rho} = 0$

→ $N = 0 \Rightarrow F_{roce} = 0$

luego en ① queda $0 = 0 + m\Omega^2\rho$

$\boxed{0 = \rho}$ → en el origen!

4) queremos $\rho(t)$ → asumiendo $\rho(t=0) = 0$

$\dot{\rho}(t=0) = v_0$

usando ✱ en ①

$$m\ddot{\rho} = -\mu 2 m \Omega \dot{\rho} + m\Omega^2 \rho$$

$$\ddot{\rho} = -2\mu\Omega\dot{\rho} + \Omega^2\rho$$

$$\boxed{\ddot{\rho} + 2\mu\Omega\dot{\rho} - \Omega^2\rho = 0}$$

↳ edo a resolver

proponemos que la solución es tipo $\rho = Ae^{\lambda t}$ (Ansatz)

$\Downarrow$

$$\dot{\rho} = \lambda A e^{\lambda t}$$

$$\ddot{\rho} = \lambda^2 A e^{\lambda t}$$

quedamos

$$\lambda^2 \cancel{Ae^{\lambda t}} + 2\mu\Omega\lambda \cancel{Ae^{\lambda t}} - \Omega^2 \cancel{Ae^{\lambda t}} = 0$$

$$\lambda^2 + 2\mu\Omega\lambda - \Omega^2 = 0$$

$$\lambda = \frac{-2\mu\Omega \pm \sqrt{4\mu^2\Omega^2 + 4\Omega^2}}{2}$$

$$\boxed{\lambda = -\mu\Omega \pm \Omega\sqrt{\mu^2 + 1}}$$

luego hay 2 soluciones

$$\rho(t) = Ae^{\lambda_1 t} + A'e^{\lambda_2 t}$$

Como $\rho(0) \to = 0$

$$Ae^{\lambda_1 t} + A'e^{\lambda_2 t}\Big|_{t=0} = 0$$

$$A + A' = 0 \to \boxed{A = -A'}$$

$$\text{y } \rho(t) = A(e^{\lambda_1 t} - e^{\lambda_2 t})$$

falta determinar A $\to$ usamos condición inicial de velocidad $\to$ $\dot{\rho}(t=0) = v_0$

$$A(\lambda_1 e^{\lambda_1 t} - \lambda_2 e^{\lambda_2 t})\Big|_{t=0} = v_0$$

$$A(\lambda_1 - \lambda_2) = v_0$$

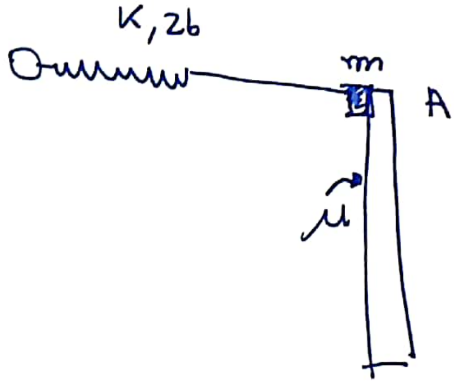
$$A = \frac{v_0}{\lambda_1 - \lambda_2} = \frac{v_0}{-\mu\Omega + \sqrt{\mu^2+1} - (-\mu\Omega - \sqrt{\mu^2+1})}$$

Escaneado con CamScanner

$$A = \frac{v_0}{2\Omega\sqrt{\mu^2+1}} \Rightarrow \boxed{\rho(t) = \frac{v_0}{2\Omega\sqrt{\mu^2+1}}\left(e^{(\mu\Omega t + \Omega\sqrt{\mu^2+1})t} - e^{(-\mu\Omega t - \Omega\sqrt{\mu^2+1})t}\right)}$$

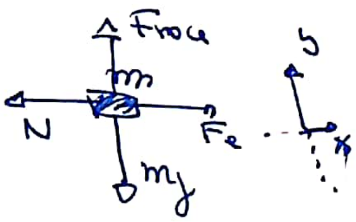
P2)

Tenemos inicialmente



a) la condición para que m pueda descender:

hacemos DCL



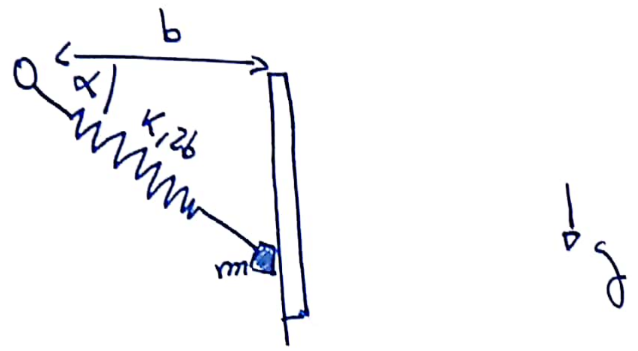
$m\ddot{y} = F_{roce} - mg$

$m\ddot{y} = \mu N - mg \Rightarrow$ para que pueda descender

$|F_{roce}| < |Peso|$

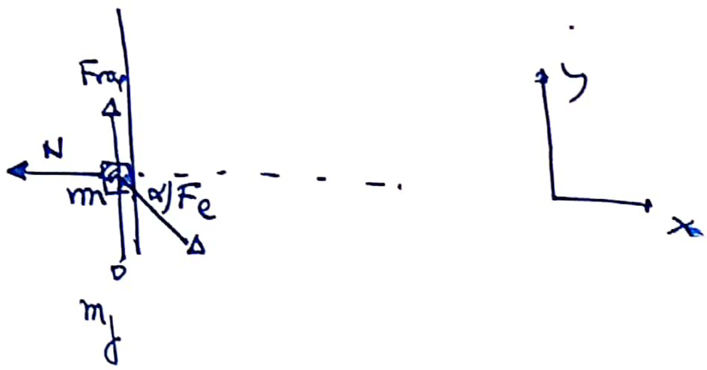
$\boxed{\mu N < mg}$

Ahora la partícula es liberada.



queremos saber $N(\alpha)$ y encontrar α de despegue

DCL



→ suma de Fuerzas:

$$\overset{\text{solo baja}}{m\cancel{\ddot{x}}^{\,0}} = Fe\cos\alpha - N \Rightarrow \boxed{N = Fe\cos\alpha}$$

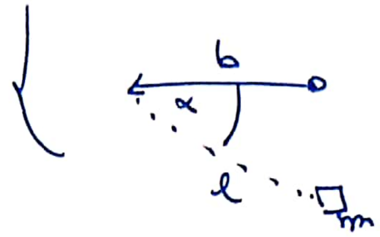
$$m\ddot{y} = Fro - mg - Fe\,\text{sen}\,\alpha$$

donde

$Fe = -k(l - 2b)$

cuando mide desde O hasta m.

$l = \dfrac{b}{\cos\alpha}$



$\longrightarrow$ $F_e - K\left(\dfrac{b}{\cos\alpha} - 2b\right) = K\left(2b - \dfrac{b}{\cos\alpha}\right)$

luego $N = K(2b - b/\cos\alpha) \cdot \cos\alpha$

$$\boxed{N = K(2b\cos\alpha - b)}$$

Cuando se despega $N = 0 \Rightarrow$ $0 = K(2b\cos\alpha - b)$

$0 = 2\cos\alpha - 1$

$\frac{1}{2} = \cos\alpha$

$\cos^{-1}(1/2) = \alpha$

$60° = \alpha$

$$\boxed{\frac{\pi}{3} = \alpha}$$

Ahora queremos la velocidad de despegue.

para eso $\longrightarrow$ $E_f - E_0 = W$

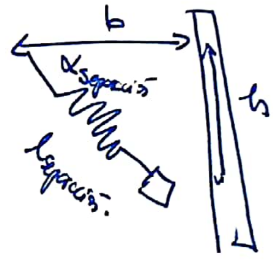
cinetica — potencial — elástica

$$E_0 = K_0 + U_{g0} + U_{e0}$$

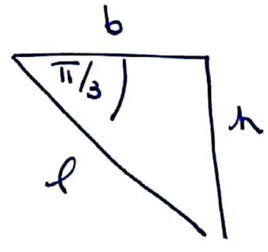
$$= 0 + \underbrace{0}_{\text{es 0 ya que tomamos ese origen!}} + \underbrace{\frac{k(b-2b)^2}{2}}_{\frac{kb^2}{2}}$$

$$E_f = K_f + U_{gf} + U_{ef}$$

$$= \frac{mv^2}{2} - mgh + \frac{k}{2}(l-2b)^2$$



ya sabemos que $\underline{\alpha = \pi/3}$



$$\tan(\pi/3) = \frac{h}{b}$$

$$b\tan(\pi/3) = h$$

$$\boxed{b \cdot \sqrt{3} = h}$$

$$\frac{b}{l} = \cos(\pi/3)$$

$$\boxed{l = \frac{b}{\cos(\pi/3)} = 2b}$$

luego

$$E_f = \frac{mv^2}{2} - mgb\sqrt{3} + \frac{K}{2}\cancel{(2b-2b)}^{2}$$

$$\longrightarrow \quad E_f - E_0 = W$$

$$\frac{mv^2}{2} - \sqrt{3}mgb - \frac{K}{2}b^2 = W$$

$$\boxed{v^2 = \left(W + \sqrt{3}\,mgb + \frac{Kb^2}{2}\right)\cdot\frac{2}{m}}$$

hay que encontrar el trabajo hecho por el roce!

$$W = \int F_{roce} \cdot dr$$

$$= \int_0^{-h} \mu N \, dy$$

$$= \mu \int_0^{-h} Kb(2\cos\alpha - 1)\, dy$$

como vimos antes

$N = Kb(2\cos\alpha - 1)$

$$= \mu \int_0^{-h} Kb(2\cos\alpha)\,dy - \mu \int_0^{-h} Kb\, dy$$

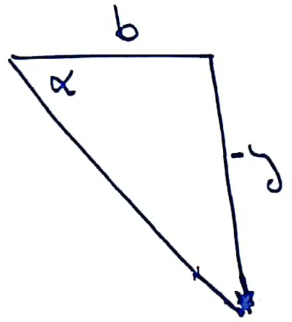
$$= \mu K b \, 2 \int_0^{-h} \cos\alpha \, dy \; - \; \underbrace{\mu K b \int_0^{-h} dy}$$

$$- \big(- \underbrace{\mu K b \cdot h} \big)$$

$$- \big(- \mu K b \cdot (b \sqrt{3}) \big)$$

Solo debemos ahora encontrar la integral $\int_0^{-h} \cos\alpha \, dy$

hay que encontrar una relación entre α e y:



$$\tan\alpha = \frac{-y}{b}$$

Aquí hay 2 opciones, o intentamos resolver la integral, usando $\alpha = \arctan(-y/b)$

$$\longrightarrow \int_0^{-h} \cos(\arctan(-y/b)) \, dy$$

que se ve difícil,

o encontrar una relación entre dy y $d\alpha$ e integrar en α :

$\tan(\alpha) = -y/b \qquad (d(\ ))$

· $d(\tan(\alpha)) = \dfrac{-dy}{b}$

$$\frac{1}{\cos^2\alpha}\, d\alpha = \frac{-dy}{b} \longrightarrow \underbrace{dy = \frac{-b\, d\alpha}{\cos^2\alpha}}$$

con este cambio la integral queda:

$$\underbrace{\int_{\alpha(y=0)}^{\alpha(y=-h)} \frac{\cos\alpha \cdot (-b)\, d\alpha}{\cos^2\alpha}}$$

$$\int_0^{\pi/3} \frac{-b\, d\alpha}{\cos\alpha}$$

$$\underbrace{-b \int_0^{\pi/3} \frac{d\alpha}{\cos\alpha}}$$

$$-b\left(\ln\left(\tan\left(\pi/4 + \frac{\alpha}{2}\right)\right) \Big/_{0}^{\pi/3}\right)$$

$$-b\left(\ln\left(\tan(\pi/4 + \pi/6) \big/ \cancelto{1}{\tan(\pi/4)}\right)\right)$$

$$-b \ln(\tan(\pi/4 + \pi/6))$$

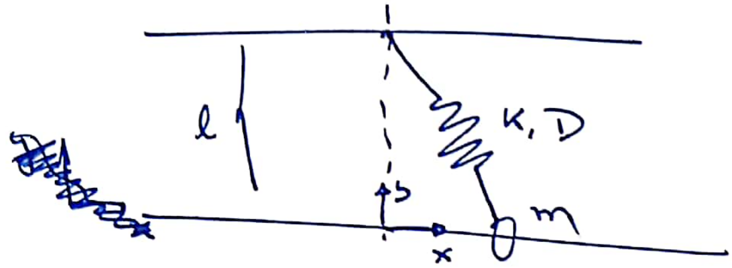
Luego $W = \mu K b 2\left(-b \ln\left(\tan\left(\pi/4 + \pi/6\right)\right)\right) + \mu K b\, b\sqrt{3}$

$$= \mu K b^2\left(-2\ln\left(\tan\left(\pi/4 + \pi/6\right)\right) + \sqrt{3}\right)$$

$$\boxed{\frac{m v^2}{2} = W + \sqrt{3}\, m g b + \frac{K b^2}{2}}$$

P3)

Tenemos



tomamos referencia en el origen

1) queremos $U(x) = \cancel{U_g}^{\,0} + U_e$

$= U_e$

$= \frac{K}{2}\left(\text{largo} - D\right)^2$

largo $= \sqrt{l^2 + x^2}$ $\quad = \frac{K}{2}\left(\sqrt{l^2+x^2} - D\right)^2$

$$\boxed{U = \frac{K}{2}\left(\sqrt{l^2+x^2} - D\right)^2}$$

Tenemos que Energía = cte

$U + K = \text{cte}$

$$\frac{K}{2}\left(\sqrt{l^2+x^2}-D\right)^2 + \frac{m\dot{x}^3}{2} = cte \;\Big/ \frac{d}{dt}$$

$$\frac{K}{2}\left(\sqrt{l^2+x^2}-D\right)^{2-1} \cdot \frac{1}{\cancel{2}\sqrt{l^2+x^2}} \cdot \cancel{2}x\cdot\cancel{\dot{x}} + \frac{\cancel{2}m\cancel{\dot{x}}\ddot{x}}{\cancel{2}} = 0$$

$$\boxed{-\frac{Kx\left(\sqrt{l^2+x^2}-D\right)}{2\sqrt{l^2+x^2}} = m\ddot{x}}$$

recuperamos ecuación de newton!

equilibrios $\Longrightarrow$ $\ddot{x} = 0$

$$-Kx\left(\sqrt{l^2+x^2}-D\right) = 0$$

$$\boxed{x = 0}$$

$$\sqrt{l^2+x^2} = D$$

$$\boxed{x = \pm\sqrt{D^2-l^2}}$$