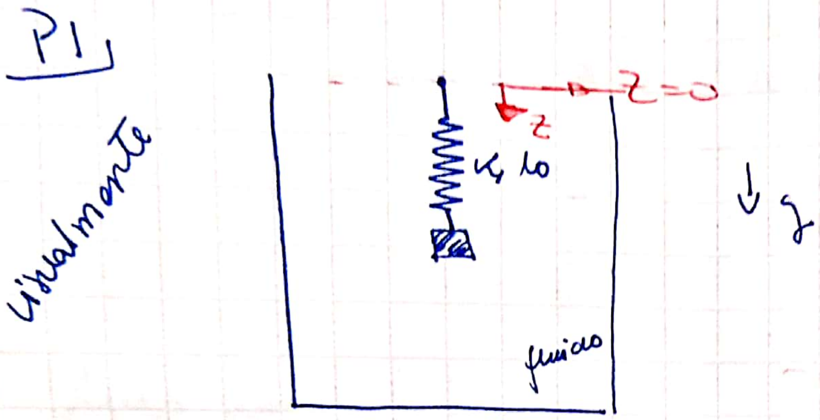
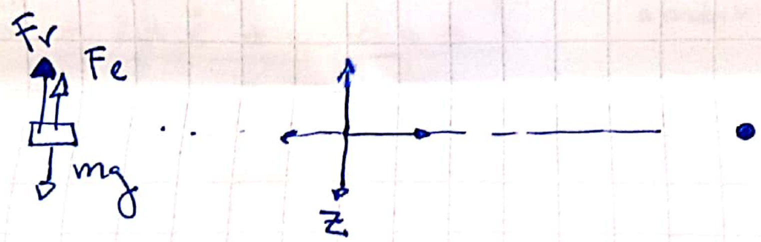
Pauta Aux 11

P1



DCL



escribimos la ecuación:

$$m\ddot{z} = \overbrace{-k(z - l_0)}^{\text{resorte}} - \overbrace{c\dot{z}}^{\text{roce}} + mg$$

$\omega^2 = \frac{k}{m}$

$$\ddot{z} = -\frac{k}{m}(z - l_0) - \frac{c}{m}\dot{z} + g$$

$2\gamma = \frac{c}{m}$

$$\ddot{z} = -\omega^2 (z - l_0) - 2\gamma\dot{z} + g$$

$$\ddot{z} + 2\gamma\dot{z} + \omega^2(z - l_0) - g = 0$$

para que nos quede exactamente la ecuación amortiguada:

cambiamos de variable (para que quede EDO homogénea)

$$x = z - (l_0 + g/\omega^2)$$

$$\dot{x} = \dot{z}$$

$$\ddot{x} = \ddot{z}$$

Luego queda:

$$\boxed{\ddot{x} + 2\mu\dot{x} + \omega^2 x = 0}$$ → ecuación oscilador amortiguado

¿por qué $x = z - (l_0 + g/\omega^2)$?

El término que molesta es $\omega^2(z - l_0) - g$

$$\omega^2(z - l_0 - g/\omega^2)$$

$$\omega^2(\underbrace{z - (l_0 + g/\omega^2)}_{x})$$

→ Decimos que solución es tipo

$$x(t) = Ae^{\lambda t}$$

$$\to \cancel{A}\lambda^2 \cancel{e^{\lambda t}} + 2\mu \cancel{A}\lambda \cancel{e^{\lambda t}} + \omega^2 \cancel{A e^{\lambda t}} = 0$$

$$\lambda^2 + 2\mu\lambda + \omega^2 = 0 \Rightarrow \lambda = \frac{-2\mu \pm \sqrt{(2\mu)^2 - 4\omega^2}}{2}$$

$$\boxed{\lambda = -\mu \pm \sqrt{\mu^2 - \omega^2}}$$

luego $x(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$

$$= Ae^{-\gamma + \sqrt{\gamma^2-\omega^2}\,t} + Be^{-\gamma - \sqrt{\gamma^2-\omega^2}\,t}$$

$$\boxed{x(t) = e^{-\gamma t}\cdot\left(Ae^{\sqrt{\gamma^2-\omega^2}\,t} + Be^{-\sqrt{\gamma^2-\omega^2}\,t}\right)}$$

Buscamos las constantes si $y(0) = H$, $\dot{y}(0) = v_0$

$$H = 1\cdot(A+B) \rightarrow \boxed{H = A+B} \quad ①$$

$$\dot{y} = -\gamma e^{-\gamma t}\left(Ae^{\sqrt{\gamma^2-\omega^2}\,t} + Be^{-\sqrt{\gamma^2-\omega^2}\,t}\right) + e^{-\gamma t}\left(A\sqrt{\gamma^2-\omega^2}\,e^{\sqrt{\gamma^2-\omega^2}\,t} - B\sqrt{\gamma^2-\omega^2}\,e^{-\sqrt{\gamma^2-\omega^2}\,t}\right)$$

$\cancel{v_0}$

$$v_0 = -\gamma(A+B) + 1\left(A\sqrt{\gamma^2-\omega^2} - B\sqrt{\gamma^2-\omega^2}\right)$$

$$\boxed{v_0 = -A\left(\gamma - \sqrt{\gamma^2-\omega^2}\right) - B\left(\gamma^2 + \sqrt{\gamma^2-\omega^2}\right)} \quad ②$$

en ① $\rightarrow$ $B = H - A$ $\rightarrow$ $\boxed{B = \dfrac{-v_0 - H\left(\gamma - \sqrt{\gamma^2-\omega^2}\right)}{2\sqrt{\gamma^2-\omega^2}}}$

en ② $\rightarrow$ $\boxed{A = \dfrac{v_0 + H\left(\gamma + \sqrt{\gamma^2-\omega^2}\right)}{2\sqrt{\gamma^2-\omega^2}}}$

Queremos alguna condición $\longrightarrow$ imponemos por [illegible]

$$\longrightarrow \quad x_{sg}(t) = 0$$

$$e^{-\gamma T}\left(A e^{\sqrt{\gamma^2-\omega^2}\,T} + B e^{-\sqrt{\gamma^2-\omega^2}\,T}\right) = 0$$

$$A e^{\sqrt{\gamma^2-\omega^2}\,t} = -B e^{-\sqrt{\gamma^2-\omega^2}\,t}$$

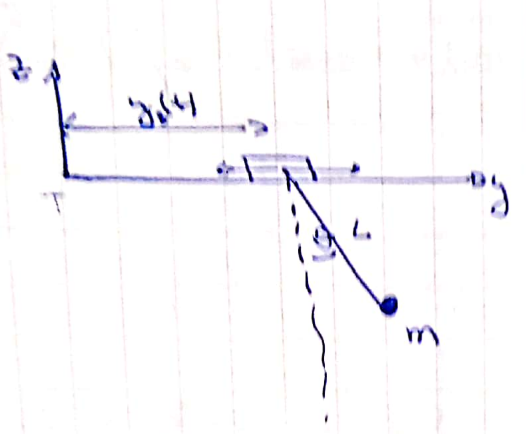
$$e^{2\sqrt{\gamma^2-\omega^2}\,T} = -\frac{B}{A}$$

$$2\sqrt{\gamma^2-\omega^2}\,T = \ln\left(-\frac{B}{A}\right)$$

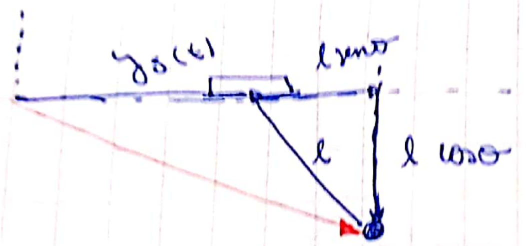
$$T = \frac{\ln(-B/A)}{2\sqrt{\gamma^2-\omega^2}}$$

Para que exista $T > 0$

P2)



Describiremos el movimiento: para ello necesitamos un vector posición



$$\vec{r} = (y_s(t) + l\sin\theta)\,\hat{y} - l\cos\theta\,\hat{z}$$

$$m\ddot{\vec{r}} = m(\ddot{y}_s(t) + (\ddot{l\sin\theta}))\,\hat{y} - (\ddot{l\cos\theta})\,\hat{z}$$

$$\to l\cos\theta\,\dot{\theta}^2 + l\sin\theta\,\ddot{\theta}$$

$$\frac{d}{dt}(l\sin\theta) = l\cos\theta\,\dot{\theta}$$

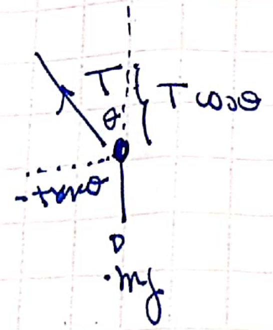
$$\frac{d}{dt}(l\cos\theta\,\dot{\theta}) = (-l\sin\theta\,\dot{\theta}^2 + l\cos\theta\,\ddot{\theta})$$

luego $\ddot{\vec{r}} = \underbrace{(\ddot{y}_s - l\,\text{sen}\theta\,\dot{\theta}^2 + l\cos\theta\,\ddot{\theta})}_{\ddot{y}}\hat{y} + \underbrace{(l\cos\theta\,\dot{\theta}^2 + l\,\text{sen}\theta\,\ddot{\theta})}_{\ddot{z}}\hat{z}$

luego $m\ddot{y} = \sum F_y = ¿?$

$m\ddot{z} = \sum F_z = ¿?$

las fuerzas del problema son T y mg



$$\left.\begin{array}{l} \rightarrow m\ddot{y} = -T\,\text{sen}\theta \\ m\ddot{z} = -mg + T\cos\theta \end{array}\right\} \text{queremos encontrar } \ddot{\theta} = \text{algo}$$

$\underline{y)}$ $m(\ddot{y}_s(t) + l\cos\theta\,\ddot{\theta} - l\,\text{sen}\,\theta\,\dot{\theta}^2) = -T\,\text{sen}\,\theta$

$\underline{z)}$ $m(l\,\text{sen}\,\theta\,\ddot{\theta} + l\cos\theta\,\dot{\theta}^2) = T\cos\theta - mg$

→ despejamos $\dot{\theta}^2$ de alguna ecuación, y luego obtendremos $\ddot{\theta} = f(\theta)$

de $y)$ → $m\,l\,\text{sen}\,\theta\,\dot{\theta}^2 = T\,\text{sen}\,\theta + m(\ddot{y}_s + l\cos\theta\,\ddot{\theta})$

$$\dot{\theta}^2 = \frac{T}{ml} + \frac{\ddot{y}_s}{l\,\text{sen}\,\theta} + \frac{\cos\theta\,\ddot{\theta}}{\text{sen}\,\theta}$$

ahora en $\underline{z)}$

$$m\left(l\,\text{sen}\,\theta\,\ddot{\theta} + l\cos\theta\left(\cancel{\frac{T}{ml}} + \frac{\ddot{y}_s}{l\,\text{sen}\,\theta} + \frac{\cos\theta\,\ddot{\theta}}{\text{sen}\,\theta}\right)\right) = \cancel{T\cos\theta} - mg$$

$$m\left(l\,\text{sen}\,\theta\,\ddot{\theta} + \ddot{y}_s\frac{\cos\theta}{\text{sen}\,\theta} + \frac{l\cos^2\theta\,\ddot{\theta}}{\text{sen}\,\theta}\right) = -mg \qquad / \cdot \text{sen}\,\theta$$

$$m(l\,\text{sen}^2\theta\,\ddot{\theta} + l\cos^2\theta\,\ddot{\theta} + \ddot{y}_s\cos\theta) = -mg\,\text{sen}\,\theta$$

$$\cancel{m}\,l\,\ddot{\theta} + \cancel{m}\,\ddot{y}_s\cos\theta = -\cancel{m}\,g\,\text{sen}\,\theta$$

$$\boxed{\ddot{\theta} + \frac{g}{l}\operatorname{sen}\theta = -\ddot{y}_s\frac{\cos\theta}{l}}$$

en pequeñas oscilaciones:

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{-\ddot{y}_s}{l}$$

$$\boxed{\ddot{\theta} + \omega^2\theta = -\frac{\ddot{y}_s}{l}}$$

Ahora si $y_s = y_0 \cos(\Omega t)$

$$\dot{y}_s = -y_0 \Omega \operatorname{sen}(\Omega t)$$

$$\ddot{y}_s = -y_0 \Omega^2 \cos(\Omega t)$$

luego

$$\ddot{\theta} + \omega^2\theta = \underbrace{\frac{y_0\Omega^2\cos(\Omega t)}{l}}_{\text{forzamiento!}}$$

para solucionar:

Escaneado con CamScanner

Sol homogénea + sol particular

M.a.S

$$\theta_p = \overset{C}{cte} \cos(\Omega t)$$

$$\theta(t) = Ae^{i\omega t} + Be^{i\omega t}$$

$$\ddot{\theta}_p = -\Omega^2 C \cos(\Omega t)$$

- $\theta(t) = A\operatorname{sen}(\omega t + \phi)$

en la ecuación

- $\theta(t) = A\operatorname{sen}(\omega t) + B\cos(\omega t)$

$$-\Omega^2 C \cancel{\cos(\Omega t)} + \omega^2 C \cancel{\cos(\Omega t)} = \frac{y_0 \Omega^2}{l} \cancel{\cos(\Omega t)}$$

$$\boxed{C = \frac{y_0 \Omega^2}{l(\omega^2 - \Omega^2)}}$$

luego

$$\theta(t) = A\operatorname{sen}(\omega t + \phi) + \frac{y_0 \Omega^2}{l(\omega^2 - \Omega^2)} \cos(\Omega t)$$

resonancia si $\omega^2 = \Omega^2$

P3)

Nos dicen $U(r) = E_0 \ln(r/r_0)$

a) queremos encontrar radio r_c que tiene momento angular l.

$\longrightarrow \vec{l} = \vec{r} \times m\vec{v}$

Si describimos en polares $\left\{ \begin{array}{l} \vec{l} = r\hat{r} \times m(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta}) \\ \boxed{\vec{l} = m r^2 \dot{\theta} \, \hat{z}} \end{array} \right.$

$\hookrightarrow$ hay que encontrar $\dot{\theta}$

como $U(r)$, es fuerza central $\longrightarrow -\frac{dU}{dr} = F$

$$\boxed{-\frac{E_0}{r}\hat{r} = \vec{F}}$$

tomemos entonces $ma = F$

$$m\left((\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} + \ddot{z}\hat{z}\right) = -\frac{E_0}{r}\hat{r}$$

$\longrightarrow$ en $\hat{r}$, $\boxed{m(\ddot{r} - r\dot{\theta}^2) = \frac{-E_0}{r}}$ en $\hat{\theta}$ $\boxed{2\dot{r}\dot{\theta} + r\ddot{\theta} = 0}$

en $\hat{z}$ $\ddot{z} = 0$

como se mueve circularmente $\dot{r} = \ddot{r} = 0$ y $r = r_c$ (menos)

$$\rightarrow \boxed{-\frac{E_0}{r_c} = -m r_c \dot{\theta}^2} \quad \hat{r}$$

$$\rightarrow \boxed{\dot{\theta}^2 = \frac{E_0}{m r_c^2}}$$

Luego como $l = m r^2 \dot{\theta}$

$$= m r_c^2 \left(\frac{E_0}{m r_c^2}\right)^{1/2}$$

$$l = m\, r_c \left(\frac{E_0}{m}\right)^{1/2}$$

$$l = r_c (m E_0)^{1/2} \longrightarrow \boxed{r_c = \frac{l}{\sqrt{m E_0}}}$$

b) ahora para ω^2 vamos "perturbamos".

$$\rightarrow E = \frac{m v^2}{2} + U(r) = \frac{m}{2}(\dot{r}^2 + r^2 \dot{\theta}^2) + E_0 \ln(r/r_0)$$

como $l = m r^2 \dot{\theta} \rightarrow \dot{\theta} = \frac{l}{m r^2}$

$$E = \frac{m\dot{r}^2}{2} + \frac{m}{2}\frac{r^2\ell^2}{m^2 r^4} + E_0 \ln(r/r_0)$$

$$\boxed{E = \frac{m\dot{r}^2}{2} + \frac{\ell^2}{2mr^2} + E_0 \ln(r/r_0)}$$

solo dependientes de r!

llamamos una expresa $E(r) = \frac{\alpha \dot{r}^2}{2} + U_{eff}(r) \longrightarrow \boxed{\omega_p^2 = \frac{U''_{eff}(r_{eq})}{\alpha}}$

en este caso $\boxed{U_{eff}(r) = \frac{\ell^2}{2mr^2} + E_0 \ln(r/r_0)}$

$r_{eq} = r_c$ y $\alpha = m$

$$\longrightarrow \omega_p^2 = \frac{\left(\ell^2/2mr^2 + E_0 \ln(r/r_0)\right)''\Big/_{r_c}}{m}$$

$$= \frac{\left(-\frac{E_0}{r^2} + \frac{3\ell^2}{mr^4}\right)\Big/_{r_c}}{m} = \boxed{\frac{2E_0^2}{\ell^2} = \omega_p^2}$$