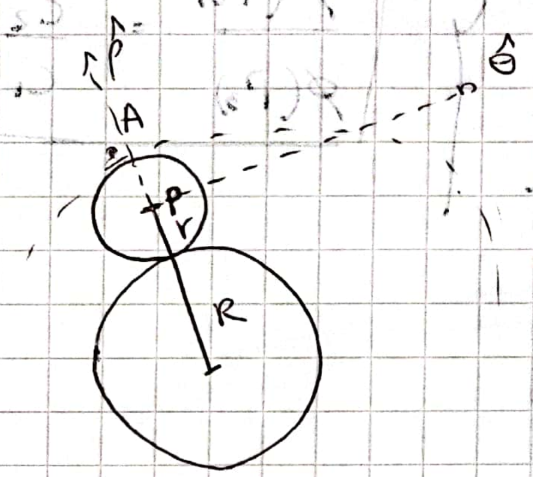
Mecanica

Pauta Aux 13

P3



Notemos primero que $\vec{V}_P = (R+r)\,\dot{\theta}\,\hat{\theta}$

$\dot{\theta}$: vel. angular de la barra (dato)

Por otra parte por ~~def~~

$$\underbrace{\vec{V}_A}_{\text{vel punto A}} = \underbrace{\vec{V}_P}_{\text{vel punto P}} + \underbrace{\vec{\omega}}_{\text{vel. angular de circulo de radio } r} \times \vec{PA}$$

pero A no se mueve $\rightarrow$ $V_A = 0$

$$\rightarrow \vec{V}_P = -\vec{\omega} \times \vec{PA}$$

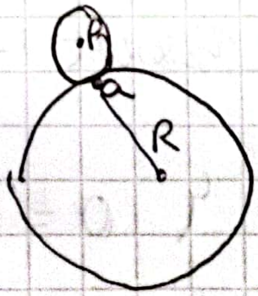
$$= -\omega \hat{k} \times r\hat{r}$$

$$\vec{V}_P = -\omega r \hat{\theta}$$

luego $(R+r)\dot{\theta} = -\omega r \rightarrow \underbrace{\omega}_{\omega \text{ de } (3)} = \frac{-(R+r)}{r}\dot{\theta}$

Pero buscamos ω_2

tomemos un punto a entre (2) y (3)



$$\vec{V}_a = \vec{V}_P + \vec{\omega} \times \vec{Pa}$$

$$= \vec{V}_P + \omega\hat{k} \times (-r\hat{r})$$

$$= \vec{V}_P + \underbrace{-\omega r \hat{r}}_{= \vec{V}_P}$$

$$= 2\vec{V}_P$$

$$\boxed{\vec{V}_a = 2(r+R)\dot{\theta}}$$

pero $\vec{V}_a = \omega_2 \cdot R\hat{\theta}$

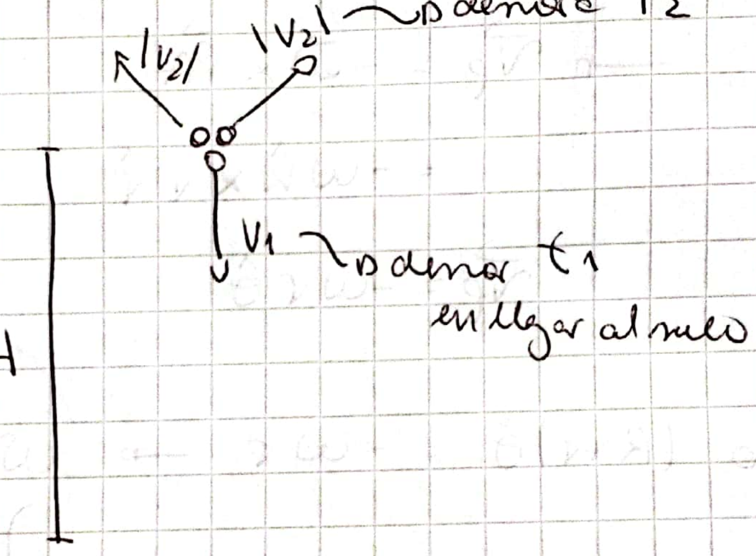
igualando

$$\boxed{\omega_2 = \frac{2(r+R)}{R}\dot{\theta}}$$

$$\boxed{\dot{\omega}_2 = \frac{2(r+R)}{R}\ddot{\theta}}$$

P1)

Terrena



$\rightarrow y_1(t) = H - v_1 t - g t^2/2$

$y_2(t) = H + v_2 \operatorname{sen}\theta\, t - g t^2/2$

y en $t_1 \longrightarrow y_1 = 0 \quad \{ \quad 0 = H - v_1 t_1 - g t_1^2/2$

$t_2 \longrightarrow y_2 = 0 \quad \{ \quad 0 = H + v_2 \operatorname{sen}\theta\, t_2 - g \frac{t_2^2}{2}$

Antes de explotar $\vec{V}_{cm} = 0$

$\rightarrow \vec{P}_{cm} = 0$ (se conserva)

$0 = m\vec{V_1} + m\vec{V_2}^1 + m\vec{V_2}^2$

$0 = -V_1\hat{y} + (\cancel{V_2\cos\theta\hat{x}} + V_2 \operatorname{sen}\theta\hat{y})$

$+ (\cancel{-V_2\cos\theta\hat{x}} + V_2\operatorname{sen}\theta\hat{y})$

$$\boxed{V_1 = 2V_2\operatorname{sen}\theta}$$

Usando esto tenemos

$0 = H - 2V_2\operatorname{sen}\theta\, t_1 - g\frac{t_1^2}{2}$ ①

$0 = H + V_2\operatorname{sen}\theta\, t_2 - g\frac{t_2^2}{2}$ ②

luego ① $\cdot t_2 - 2\cdot$ ② t_1

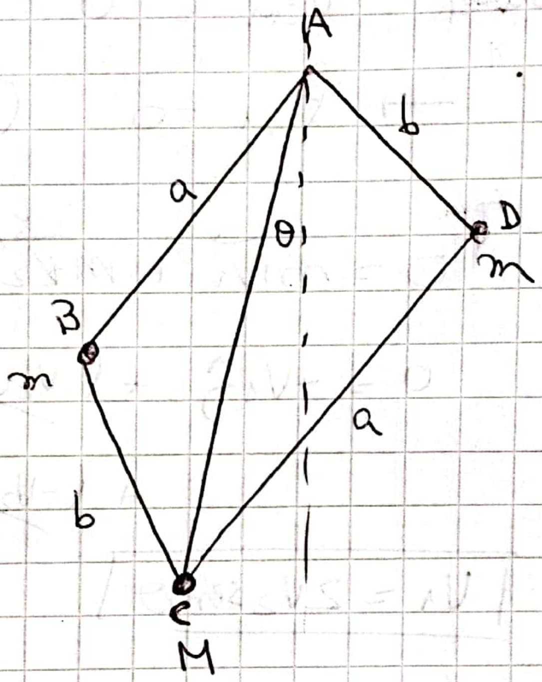
$0 = Ht_2 - \cancel{2V_2\operatorname{sen}\theta\, t_1t_2} - g\frac{t_1^2t_2}{2} - 2Ht_1 + \cancel{2V_2\operatorname{sen}\theta\, t_1t_2} + g\frac{t_1t_2^2}{2}$

$$\boxed{H = \frac{g}{2}\frac{t_1t_2(t_2-t_1)}{t_2-2t_1}}$$

u

P2

Tenemos:



Primero queremos encontrar θ_{eq} y ω

Para ello lo que debemos hacer es encontrar el Torque del centro de masa y su momentum angular

Luego $\underbrace{\tau_{CM} = \dot{l}}$

de aquí saldrá una ec. tipo resorte.

Para ello primero hay que encontrar $\vec{R}_{CM}$

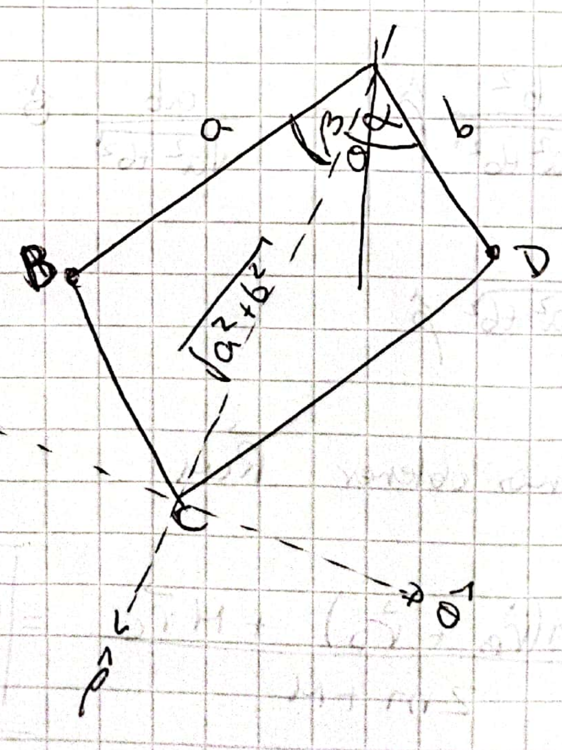
Encontramos $\vec{R}_{CM}$, para ello

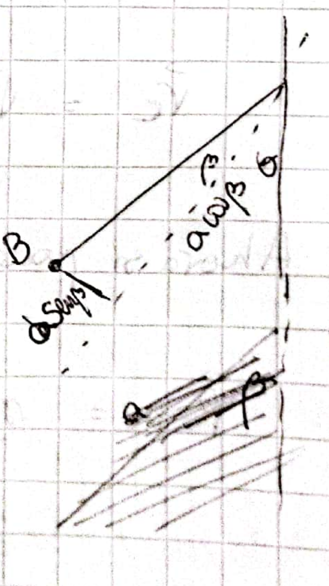
$$\vec{R}_{CM} = \frac{m\vec{r}_D + m\vec{r}_B + M\vec{r}_C}{M+2m}$$

Encontramos r_C, r_D, r_B

Definimos polares tal que $\hat{\rho}$ indique a C (así estudiamos θ)

$$\left.\begin{aligned}\vec{r}_C &= \sqrt{a^2+b^2}\,\hat{\rho} \\ \vec{r}_B &= a\cos\beta\,\hat{\rho} \\ &\quad + -a\,\mathrm{sen}\beta\,\hat{\theta} \\ \vec{r}_D &= b\cos\alpha\,\hat{\rho} \\ &\quad + b\,\mathrm{sen}\alpha\,\hat{\theta}\end{aligned}\right\}$$





Pero $\operatorname{sen}\alpha = \frac{a}{\sqrt{a^2+b^2}}$ $\quad \cos\alpha = \frac{b}{\sqrt{a^2+b^2}}$

$\operatorname{sen}\beta = \frac{b}{\sqrt{a^2+b^2}}$ $\quad \cos\beta = \frac{a}{\sqrt{a^2+b^2}}$

Luego

$$\vec{r}_B = \frac{a^2}{\sqrt{a^2+b^2}}\hat{\rho} - \frac{ab}{\sqrt{a^2+b^2}}\hat{\theta}$$

$$\vec{r}_P = \frac{b^2}{\sqrt{a^2+b^2}}\hat{\rho} + \frac{ab}{\sqrt{a^2+b^2}}\hat{\theta}$$

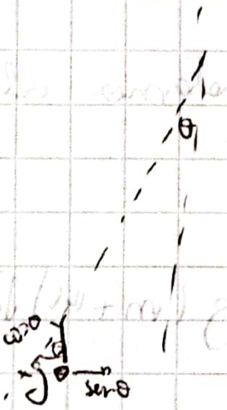
$$\vec{r}_C = \sqrt{a^2+b^2}\,\hat{\rho}$$

Ahora si podemos obtener $\vec{R}_{CM}$

$$\vec{R}_{CM} = \frac{m(\vec{r}_B + \vec{r}_P) + M\vec{r}_C}{2m+M} = \boxed{\frac{(m+M)\sqrt{a^2+b^2}}{M+2m}\hat{\rho}}$$

Luego $\tau_{CM} = \vec{R}_{CM} \times (m+m+M)\vec{g}$

donde $\vec{g} = g(\cos\theta\,\hat{\rho} - \sin\theta\,\hat{\theta})$



luego

$$\tau_{CM} = \frac{(m+M)}{\cancel{M+2m}}\sqrt{a^2+b^2}\,\hat{\rho} \times (\cancel{2m+M})g(\cos\theta\,\hat{\rho} - \sin\theta\,\hat{\theta})$$

$$= (m+M)g\sqrt{a^2+b^2}\left(\overset{0}{\cancel{\hat{\rho}\times\cos\theta\,\hat{\rho}}} - \hat{\rho}\times\sin\theta\,\hat{\theta}\right)$$

$$\boxed{\tau_{CM} = -(m+M)g\sqrt{a^2+b^2}\,\sin\theta\,\overset{\hat{k}}{\cancel{(\hat{\rho}\times\hat{\theta})}}}$$

Ahora igualamos a $\dot{\vec{l}}$ tendremos una ecuación tipo resorte.

$$\vec{l} = \sum \vec{l}_i = \vec{l}_B + \vec{l}_C + \vec{l}_D$$

$$= mb^2\dot{\theta}\,\hat{k} + M(a^2+b^2)\dot{\theta}\,\hat{k} + ma^2\dot{\theta}\,\hat{k}$$

$$\vec{l} = (M+m)(a^2+b^2)\dot{\theta}\,\hat{k}$$

luego $\vec{l} = (m+M)(a^2+b^2)\dot{\theta}\,\hat{k}$

igualamos al τ_{cm}:

$$-g(m+M)\sqrt{a^2+b^2}\,\operatorname{sen}\theta = (m+M)(a^2+b^2)\ddot{\theta}$$

$$\boxed{\ddot{\theta} = \frac{-g}{\sqrt{a^2+b^2}}\operatorname{sen}\theta}$$

luego $\theta_{ex} = 0$ y $\omega = \dfrac{-g}{\sqrt{a^2+b^2}}$

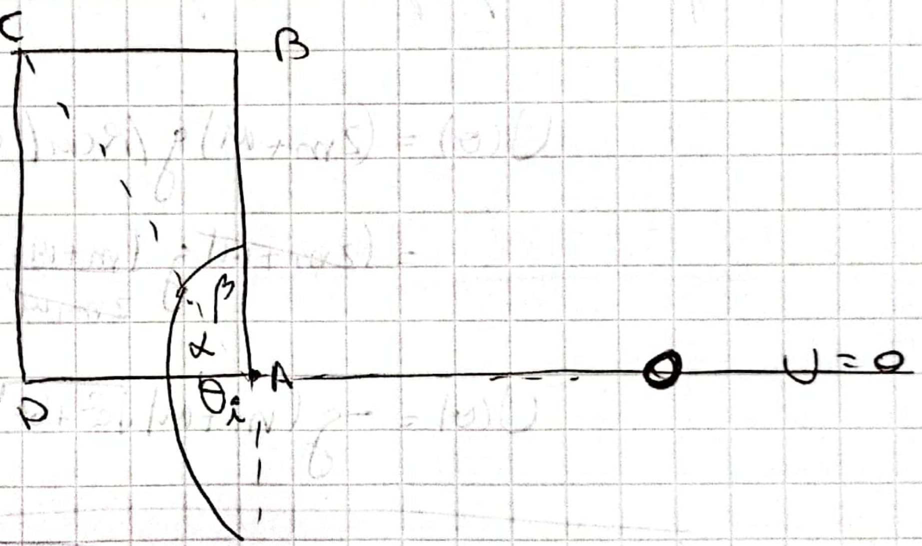
$\boxed{\ddot{\theta} = 0}$

tambien $\theta_{ex} = \pi$ pero es inestable!

b) Ahora nos preguntamos si la estructura es liberada cuando B justo verticalmente sobre A. cuál es el ángulo θ_f cuando se detiene nuevamente.

Usaremos conservación de energía del CM.

$\underline{t=0)}$



$$E_i = U_{CM} = (2m+M)\, g\, |R_{CM}|\, \sin\alpha$$

$$= \cancel{(2m+M)}\, g\, \frac{(m+M)\sqrt{a^2+b^2}}{\cancel{(2m+M)}} \cdot \frac{\sin\alpha}{\frac{a}{\sqrt{a^2+b^2}}}$$

$$\boxed{E_i = (m+M)\, g\, a}$$

Cuando se detenga denuevo nuevamente

$$E_f = U_f \quad (\text{no hay cinetica})$$

Notemos que siempre podemos definir

$$U(\theta) = (2m+M)\, g\, |R_{cm}| \sin(\theta - \pi/2)$$

$$= \cancel{(2m+M)}\, g \frac{(m+M)}{\cancel{2m+M}} \sqrt{a^2+b^2}\ \underbrace{\sin(\theta - \pi/2)}_{-\cos\theta}$$

$$U(\theta) = -g(m+M)\sqrt{a^2+b^2}\cos\theta$$

luego en $\boxed{E_f = U_f = -g(m+M)\sqrt{a^2+b^2}\cos\theta_f}$

igualamos a E_i

$$-\cancel{g(m+M)}\sqrt{a^2+b^2}\cos\theta_f = a\cancel{g(m+M)}$$

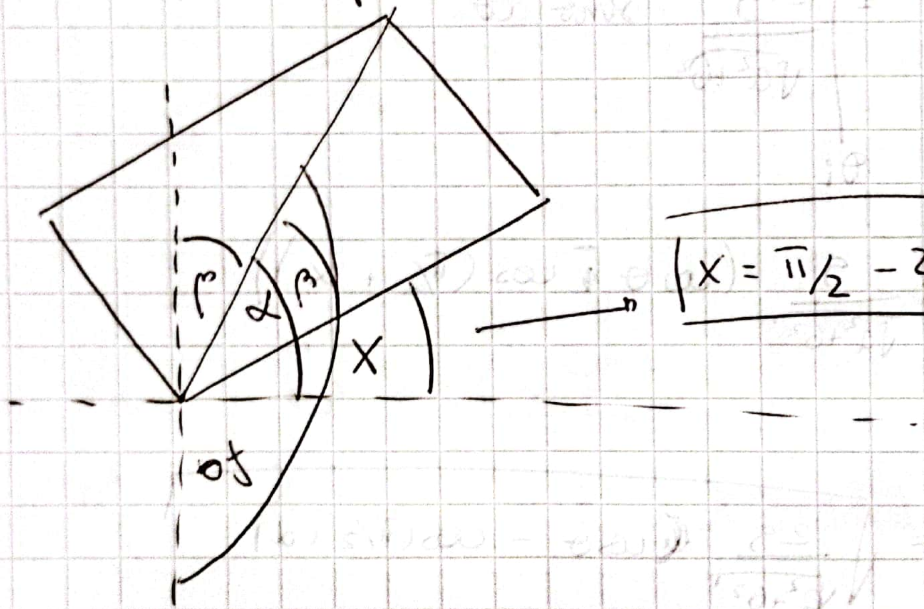
$$\cos\theta_f = \frac{-a}{\sqrt{a^2+b^2}}$$

$$\cos\theta_f = -\cos\beta \longrightarrow \theta_f = \pm(\pi - \beta)$$

Escaneado con CamScanner

$\longrightarrow \boxed{\theta_f = -\pi + \beta}$ (la otra solución da el ángulo inicial)

pero nos piden respecto de la horizontal:



$\longrightarrow \boxed{X = \pi/2 - 2\beta}$

Ahora queremos vel max de C

notemos que $\vec{V}_C = \sqrt{a^2+b^2}\ \dot{\theta}\ \hat{\theta}$

hay que encontrar $\dot{\theta}_{max}$

tenemos la ecuación de θ:

$$\ddot{\theta} = -\frac{g}{\sqrt{a^2+b^2}} \sin\theta$$

$$\dot{\theta}\frac{d\dot{\theta}}{d\theta} = \frac{-g}{\sqrt{a^2+b^2}}\operatorname{sen}\theta$$

$$\int_0^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} = \int_{\theta_i}^{\theta} \frac{-g}{\sqrt{a^2+b^2}} \operatorname{sen}\theta\, d\theta$$

$$\frac{\dot{\theta}^2}{2} = \frac{g}{\sqrt{a^2+b^2}}\left(\cos\theta - \cos\left(\pi/2 + \alpha\right)\right)$$

$$\dot{\theta} = \sqrt{\frac{2g}{\sqrt{a^2+b^2}}\left(\cos\theta - \cos(\pi/2+\alpha)\right)}$$

pero el $\dot{\theta}_{MAX}$ ocurre en $\theta = 0$ (es un péndulo)

$$\rightarrow \boxed{\dot{\theta}_{MAX} = \sqrt{\frac{2g}{\sqrt{a^2+b^2}}\left(1 - \cos\left(\pi/2 + \alpha\right)\right)}}$$