Pauta Aux 11

P1)

a) En la aux pasada se vio que

$$E = \underbrace{\frac{1}{2} m R^2 \theta^2 \dot{\theta}^2}_{E_K} + \underbrace{\frac{1}{2} \frac{m}{12} l^2 \dot{\theta}^2}_{E_\Omega} + \underbrace{mgR\left(\sqrt{1+\theta^2} - 1\right)}_{U_g}$$

El Lagrangeano es

$$L = E_c - E_p$$

$$L = \frac{1}{2} m R^2 \theta^2 \dot{\theta}^2 + \frac{1}{2} \frac{m}{12} l^2 \dot{\theta}^2 - mgR\left(\sqrt{1+\theta^2} - 1\right)$$

Luego para encontrar las ec. de mov:

$$\text{si } L(q_i, \dot{q}_i) \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i}$$

En nuestro caso $q_i = \theta$

$\dot{q}_i = \dot{\theta}$

$$\rightarrow \frac{\partial L}{\partial \theta} = \frac{1}{2} m R^2 \dot{\theta}^2 \, 2\theta \quad + \frac{1 mg R}{\cancel{2}\sqrt{1+\theta^2}} \cdot \cancel{2}\theta$$

$$= m R^2 \dot{\theta}^2 \theta \; + \frac{mg R\theta}{\sqrt{1+\theta^2}}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m R \theta^2 \, 2\dot{\theta} \quad + \frac{1}{2} \frac{m}{12} \ell^2 \, 2\dot{\theta}$$

$$= mR\theta^2 \dot{\theta} + \frac{m\ell^2}{12} \dot{\theta}$$

$$\rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = mR\left(2\theta\dot{\theta}\dot{\theta} + \theta^2 \ddot{\theta}\right) \quad + \frac{m\ell^2}{12} \ddot{\theta}$$

$$= 2mR\left(\theta\dot{\theta}^2 + \frac{\theta^2 \ddot{\theta}}{2}\right) + \frac{m\ell^2}{12} \ddot{\theta}$$

¡palomo:

$$mR^2 \cancel{\dot\theta^2}^{0}\theta - \frac{mgR\theta}{\sqrt{1+\cancel{\theta^2}_0}} = 2mR\theta\cancel{\dot\theta^2}^{0} + mR\cancel{\theta}^{2}_{0}\ddot\theta + \frac{ml^2}{12}\ddot\theta$$

$$-mgR\theta = \frac{ml^2}{12}\ddot\theta$$

$$\underbrace{-\frac{gR12}{l^2}}_{\omega^2}\theta = \ddot\theta$$

$$\boxed{-\omega^2\theta = \ddot\theta}$$

b) Se obtuvo antes que

$$K_1 = \frac{1}{2}m_1\dot y_1^2 \quad ; \quad K_2 = \frac{1}{2}m_2\dot y_2^2$$

$$U_a = \frac{1}{2}k(l^2+y_1^2)$$

$$U_b = \frac{1}{2}k(l^2+(y_1-y_2)^2)$$

$$U_c = \frac{1}{2}k(l^2+y_2^2)$$

Luego

$$L = \frac{1}{2} m_1 \dot{y}_1^2 + \frac{1}{2} m_2 \dot{y}_2^2 - \frac{1}{2} k\left(l^2 + y_1^2 \right) + \left(l^2 + (y_1 - y_2)^2 \right)$$
$$+ l^2 + y_2^2 \Big)$$

→ para masa 1 $\frac{d}{dt} \frac{\partial L}{\partial \dot{y}_1} = \frac{\partial L}{\partial y_1}$, masa 2 $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2} \right) = \frac{\partial L}{\partial y_2}$

→ $\frac{\partial L}{\partial y_1} = -\frac{1}{2} k \left(2y_1 + (y_1 - y_2) \cdot 2 \right)$

$$= -\frac{k}{2}\left(4y_1 - 2y_2 \right) = -2ky_1 + ky_2$$

$\frac{\partial L}{\partial \dot{y}_1} = m_1 \dot{y}_1 \longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1} \right) = m_1 \ddot{y}_1$

$$\Rightarrow \boxed{m_1 \ddot{y}_1 = -2ky_1 + ky_2}$$

Lo mismo para m_2

$$\frac{\partial L}{\partial y_2} = -\frac{1}{2}k\left(2y_2 + (y_1 - y_2)\cdot 2\cdot(-1)\right)$$

$$= -\frac{k}{2}\left(4y_2 - 2y_1\right) = -k2y_2 + ky_1$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) = m_2\ddot{y}_2$$

$$\longrightarrow \boxed{m_2\ddot{y}_2 = -2ky_2 + ky_1}$$

Matricialmente

$$\begin{pmatrix}\ddot{y}_1 \\ \ddot{y}_2\end{pmatrix} = -k\begin{pmatrix}\frac{2}{m_1} & \frac{-1}{m_1} \\ \frac{-1}{m_2} & \frac{2}{m_2}\end{pmatrix}\begin{pmatrix}y_1 \\ y_2\end{pmatrix}$$

c) Se obtuvo antes

$$E = \underbrace{\frac{1}{2}m(R^2\dot{\theta}^2 + R^2\,\text{sen}^2\theta\,\omega^2)}_{\text{cinética}} \underbrace{- mgR\cos\theta}_{\text{potencial}}$$

$$\Rightarrow L = \frac{1}{2}m(R^2\dot{\theta}^2 + R^2\,\text{sen}^2\theta\,\omega^2) + mgR\cos\theta$$

Luego

$$\frac{\partial L}{\partial \theta} = \frac{\cancel{2}}{\cancel{2}} m R^2 \operatorname{sen}\theta\, \omega^2 \cdot \cos\theta + mgR \cdot (-\operatorname{sen}\theta)$$

$$\frac{\partial L}{\partial \dot\theta} = \frac{1}{2} m R^2 2\dot\theta \Rightarrow \frac{d}{dt}(\ ) = mR^2\ddot\theta$$

$$\Rightarrow mR^2\ddot\theta = mR^2 \cancel{\operatorname{sen}\theta}^{\,\theta}\, \cancel{\cos\theta}\, \omega^2 - mgR\, \cancel{\operatorname{sen}\theta}^{\,\theta}$$

$$\ddot\theta = -\underbrace{\left(\frac{g}{R} - \omega^2\right)}_{}\theta$$

frecuencia de oscilación. (siempre que $\frac{g}{R} - \omega^2 > 0$

Si no, no oscila!

d) Se obtuvo que

$$E = \underbrace{\frac{1}{2} m \left(R^2\omega^2 + r^2(\omega + \dot\theta)^2 + 2Rr\omega(\omega + \dot\theta)\cos\theta\right.}_{K}$$

luego en este caso $L = E$ (ya que $U=0$)

$$\rightarrow \frac{\partial L}{\partial \theta} = \frac{1}{\cancel{2}} m \left(\cancel{2} R\omega r (\omega + \dot{\theta}) \cdot (-\operatorname{sen}\theta) \right)$$

$$= -mR\omega r(\omega + \dot{\theta}) \operatorname{sen}\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2} m \left(r^2 (\omega + \dot{\theta}) \cdot 2\dot{\theta} + 2Rr\omega \cos\theta \right)$$

$$= mr^2(\omega + \dot{\theta})\dot{\theta} + mRr\omega\cos\theta$$

$$\rightarrow \frac{d}{dt}\left(\ \right) = mr^2\left(\ddot{\theta}\dot{\theta} + (\omega + \dot{\theta})\ddot{\theta}\right) + mRr\omega \cdot (-\operatorname{sen}\theta)\dot{\theta}$$

$$= mr^2(2\dot{\theta}\ddot{\theta} + \omega\ddot{\theta}) - mRr\omega\dot{\theta}\operatorname{sen}\theta$$

igualando

$$mr^2(\cancel{2\dot{\theta}}\ddot{\theta} + \omega\ddot{\theta}) - mRr\omega\cancel{\dot{\theta}\operatorname{sen}\theta} = -mR\omega r(\omega + \cancel{\dot{\theta}})\operatorname{sen}\theta$$

$$\cancel{m}r^{\cancel{2}}\cancel{\omega}\ddot{\theta} = -\cancel{m}R\omega^{\cancel{2}}\cancel{r}\operatorname{sen}\theta$$

$$\boxed{\ddot{\theta} = -\frac{\omega R}{r}\operatorname{sen}\theta}$$