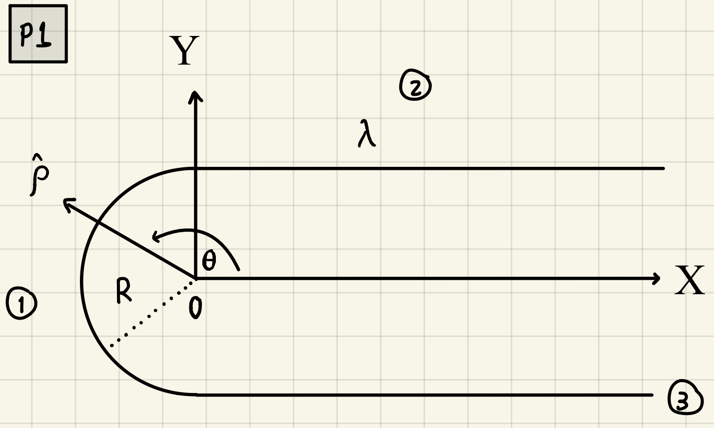
Pauta auxiliar 5

P1



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\lambda\, dl$$

campo semicircunferencia : $\vec{E}_1(\vec{r})$

- $\vec{r} = 0$
- $\vec{r}\,' = R\hat{\rho}$
- $dl = Rd\theta$
- $\theta : \frac{\pi}{2} \rightarrow \frac{3\pi}{2}$

$$\Rightarrow \vec{E}_1(0) = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{-R\hat{\rho}}{|-R|^3}\lambda\, dl. = -\frac{\lambda}{4\pi\varepsilon_0}\int \frac{R}{R^3}\hat{\rho}\cdot Rd\theta$$

$$= -\frac{\lambda}{4\pi\varepsilon_0 R}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \hat{\rho}(\theta)\, d\theta = -\frac{\lambda}{4\pi\varepsilon_0 R}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos\theta\,\hat{\imath} + \operatorname{sen}\theta\,\hat{\jmath}\, d\theta$$

* $\hat{\rho} = \cos\theta\,\hat{\imath} + \operatorname{sen}\theta\,\hat{\jmath}$

$$= -\frac{\lambda}{4\pi\varepsilon_0 R}\left(\left[\operatorname{sen}\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\hat{\imath} + \left[-\cos\theta\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}\hat{\jmath}\right)$$

$$= -\frac{\lambda}{4\pi\varepsilon_0 R}\left((-1-1)\hat{\imath} + 0\hat{\jmath}\right)$$

$$\boxed{\vec{E}_1(0) = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{\imath}}$$

campo alambre superior : $\vec{E}_2(\vec{r})$.

- $\vec{r} = 0$
- $\vec{r}\,' = x\hat{\imath} + R\hat{\jmath}$
- $dl = dx$
- $x : 0 \rightarrow \infty$

$$\Rightarrow \vec{E}_2(0) = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{-x\hat{\imath} - R\hat{\jmath}}{(x^2+R^2)^{3/2}}\cdot dx$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left[\underbrace{\int_0^\infty \frac{x\,dx}{(x^2+R^2)^{3/2}}}_{A}\hat{\imath} + R\underbrace{\int_0^\infty \frac{dx}{(x^2+R^2)^{3/2}}}_{B.}\hat{\jmath}\right]$$

B $\displaystyle\int_0^{\infty} \frac{dx}{(x^2+R^2)^{3/2}} = \frac{1}{R^3}\int_0^{\infty} \frac{dx}{\left(1+\left(\frac{x}{R}\right)^2\right)^{3/2}} = \frac{1}{R^3}\int_0^{\pi/2} \frac{1}{\left(\sec^2(u)\right)^{3/2}} \cdot \sec^2(u)\, R\, du$

$$= \frac{1}{R^2}\int_0^{\pi/2} \frac{du}{\sec(u)}.$$

cv $\left(\frac{x}{R}\right) = \tan(u).$

$\rightarrow \frac{1}{R}\,dx = \frac{du}{\cos^2(u)}. \rightarrow dx = \frac{R\,du}{\cos^2(u)}.$

- $x = 0 \rightarrow u = 0$
- $x = \infty \rightarrow u = \pi/2.$

$$= \frac{1}{R^2}\int_0^{\pi/2} \cos(u)\, du = \frac{1}{R^2}\left[\operatorname{sen}(u)\right]_0^{\pi/2}$$

$$\boxed{= \frac{1}{R^2}}$$

* $1 + \tan^2(u) = \sec^2(u).$

A $\displaystyle\int_0^{\infty} \frac{x\,dx}{(x^2+R^2)^{3/2}} = \left[-\frac{1}{\sqrt{R^2+x^2}}\right]_0^{\infty} \boxed{= \frac{1}{R}}$

$$\rightarrow \vec{E}_2(0) = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{R}\hat{\imath} + \frac{R}{R^2}\hat{\jmath}\right] = \boxed{-\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{R}\hat{\imath} + \frac{1}{R}\hat{\jmath}\right]}$$

Análogamente, para el alambre inferior: $\vec{E}_3(\vec{r})$

- $\vec{r} = 0$
- $\vec{r}' = x\hat{\imath} - R\hat{\jmath}$
- $dl = dx$
- $x: 0 \rightarrow \infty$

$$\Rightarrow \vec{E}_2(0) = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{-x\hat{\imath} + R\hat{\jmath}}{(x^2+R^2)^{3/2}} \cdot dx$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left[\underbrace{\int_0^{\infty} \frac{x\,dx}{(x^2+R^2)^{3/2}}}_{A}\hat{\imath} - R\underbrace{\int_0^{\infty} \frac{dx}{(x^2+R^2)^{3/2}}}_{B.}\hat{\jmath}\right]$$

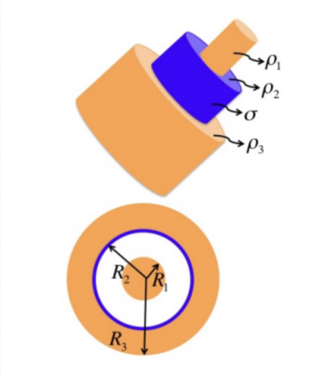
$$\boxed{\vec{E}_3(0) = -\frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{R}\hat{\imath} - \frac{1}{R}\hat{\jmath}\right]}$$

$$\therefore \vec{E}_1(0) + \vec{E}_2(0) + \vec{E}_3(0) = \frac{\lambda}{2\pi\varepsilon_0 R}\hat{\imath} - \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{R}\hat{\imath} + \frac{1}{R}\hat{\jmath}\right] - \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{R}\hat{\imath} - \frac{1}{R}\hat{\jmath}\right]$$

$$= 0 \,//$$

P2

Los cables coaxiales son utilizados para transportar señales eléctricas de alta frecuencia, por lo que son muy usados en el campo de las telecomunicaciones. Considere el cable de la figura, el cual es coaxial e infinito, compuesto por un cilindro central y diferentes casquetes cilíndricos. El primero tiene un radio R_1 y una densidad de carga ρ_1, luego viene un casquete cilíndrico con radio externo R_2 y densidad $\rho_2 = 0$, cubriéndolo, hay una placa en forma cilíndrica con densidad superficial σ y finalmente un casquete con radio externo R_3 y densidad ρ_3.



a) Encuentre el campo eléctrico en todo el espacio.

b) Encuentre el potencial eléctrico en todo el espacio (Suponga que en $V(R_0) = 0$ con R_0 conocido y $R_0 > R_3$).

c) Grafique el potencial eléctrico en función de la distancia al centro.

a) Usamos ley de Gauss: $\displaystyle\int \vec{E}\cdot d\vec{s} = \frac{Q_{enc}}{\varepsilon_0}$

Debido a la simetría cilíndrica: $\vec{E} = E(\vec{r})\,\hat{r}$

i.- $r < R_1$ $\Rightarrow$ $$E(\vec{r})\cdot\int_0^l\int_0^{2\pi} r\,d\theta\,dz = \frac{\int_0^l\int_0^{2\pi}\int_0^r \rho_1 r\,dr\,d\theta\,dz}{\varepsilon_0}$$

$$E(\vec{r})\cdot\cancel{2\pi}\cancel{r}\cancel{l} = \frac{\rho_1\cdot\frac{r^{\cancel{2}}}{2}\cdot\cancel{2\pi}\cancel{l}}{\varepsilon_0}$$

$$E(\vec{r}) = \frac{\rho_1\cdot r}{2\varepsilon_0} \Rightarrow \boxed{\vec{E}(r < R_1) = \frac{\rho_1\, r}{2\varepsilon_0}\,\hat{r}}$$

ii.- $R_1 < r < R_2$ $$\Rightarrow E(\vec{r})\cdot 2\pi r l = \frac{\int_0^l\int_0^{2\pi}\int_0^{R_1}\rho_1 r\,dr\,d\theta\,dz}{\varepsilon_0}$$ → carga encerrada es la distribuida en el volumen con radio R_1.

$$\Rightarrow E(\vec{r})\cdot\cancel{2\pi} r l = \frac{\rho_1\,\frac{R_1^2}{2}\cdot\cancel{2\pi}}{\varepsilon_0}$$

$$\Rightarrow \boxed{\vec{E}(\vec{r}) = \frac{\rho_1\,R_1^2}{2\varepsilon_0}\cdot\frac{\hat{r}}{r}}$$

iii. $R_2 < r < R_3$

$$\Rightarrow E(\vec{r})\cdot 2\pi r l = \left[\iiint_{R_2}^{r}\rho_3\,r\,dr\,d\theta\,dz + \iint\sigma R_2\,d\theta\,dz + \rho_1\,\frac{R_1^2}{2}\cdot 2\pi l\right]/\varepsilon_0$$

$$\Rightarrow E(\vec{r})\cdot\cancel{2\pi r l} = \frac{\rho_3\,\frac{r^{\cancel{2}}}{2}\cdot\cancel{2\pi l} - \rho_3\,\frac{R_2^2\,\cancel{2\pi l}}{2\,r} + \frac{\sigma}{r}R_2\,\cancel{2\pi l} + \rho_1\cdot\frac{R_1^2}{2r}\cdot\cancel{2\pi l}}{\varepsilon_0}$$

$$\Rightarrow \boxed{E(\vec{r}) = \frac{\rho_3\,r}{2\varepsilon_0} - \frac{\rho_3 R_2^2}{2r\varepsilon_0} + \frac{\sigma R_2}{r\varepsilon_0} + \frac{\rho_1 R_1^2}{2r\varepsilon_0}}$$

iv.- $R_3 < r \Rightarrow E(\vec{r}) \cdot 2\pi r L = \dfrac{\int_0^L \int_0^{2\pi} \int_{R_2}^{R_3} \rho_3 \, r \, dr \, d\theta \, dz + \sigma R_2 2\pi l + \rho_1 \frac{R_1^2}{2} 2\pi l}{\varepsilon_0}$

$$\Rightarrow E(\vec{r}) \cancel{2\pi} r \cancel{l} = \left(\rho_3 \frac{(R_3^2 - R_2^2)}{2} \cancel{2\pi l} + \sigma R_2 \cancel{2\pi l} + \rho_1 \frac{R_1^2}{2} \cancel{2\pi l} \right) / \varepsilon_0$$

$$\Rightarrow \vec{E}(\vec{r}) = \frac{\rho_3 (R_3^2 - R_2^2)}{2 \varepsilon_0 r} + \frac{\sigma R_2}{\varepsilon_0 r} + \frac{\rho_1 R_1^2}{2 \varepsilon_0 r}$$

b) como tomamos un $R_0 > R_3$, es como tomar que el potencial en el ∞ se hace 0.

iv.- $r > R_3$

$$V(r) - \underbrace{V(R_0)}_{0} = - \int_{R_0}^{r} \vec{E} \cdot d\vec{l} \Rightarrow V(\vec{r}) = - \int_{R_0}^{r} E \hat{r} \cdot dr \, \hat{r}$$

$$\Rightarrow V(\vec{r}) = - \int_{R_0}^{r} \left(\frac{\rho_3 (R_3^2 - R_2^2)}{2 \varepsilon_0 r} + \frac{\sigma R_2}{\varepsilon_0 r} + \frac{\rho_1 R_1^2}{2 \varepsilon_0 r} \right) dr$$

$$= - \frac{\rho_1 R_1^2}{2 \varepsilon_0} \ln\left(\frac{r}{R_0}\right) - \frac{\sigma R_2}{\varepsilon_0} \ln\left(\frac{r}{R_0}\right) - \frac{\rho_3 (R_3^2 - R_2^2)}{2 \varepsilon_0} \ln\left(\frac{r}{R_0}\right)$$

iii.- $R_2 < r < R_3$

$$V(\vec{r}) = - \int_{R_0}^{R_3} E(r > R_3) \cdot dr - \int_{R_3}^{r} E(R_2 < r < R_3) \cdot dr$$

$$= \underbrace{- \frac{\rho_1 R_1^2}{2 \varepsilon_0} \ln\left(\frac{R_3}{R_0}\right) - \frac{\sigma R_0}{\varepsilon_0} \ln\left(\frac{R_3}{R_0}\right) - \frac{\rho_3 (R_3^2 - R_2^2)}{2 \varepsilon_0} \ln\left(\frac{R_3}{R_0}\right)}_{C_1} - \int_{R_3}^{r} \left(\frac{\rho_3 r}{2 \varepsilon_0} - \frac{\rho_3 R_2^2}{2 r \varepsilon_0} + \frac{\sigma R_2}{\varepsilon_0 r} + \frac{\rho_1 R_1^2}{2 r \varepsilon_0} \right) dr$$

$$V(\vec{r}) = C_1 - \frac{\rho_3}{4 \varepsilon_0} (r^2 - R_3^2) - \frac{\sigma R_2}{\varepsilon_0} \cdot \ln\left(\frac{r}{R_3}\right) - \left(\frac{\rho_1 R_1^2}{2 \varepsilon_0} - \frac{\rho_3 R_2^2}{2 \varepsilon_0} \right) \cdot \ln\left(\frac{r}{R_3}\right)$$

ii.- $R_1 < r < R_2$

$$V(\vec{r}) = -\underbrace{\int_{R_0}^{R_3} E(r > R_3)\,dr}_{} - \underbrace{\int_{R_3}^{R_2} E(R_2 < r < R_3)\,dr}_{C_2} - \int_{R_2}^{r} E(R_1 < r < R_2).$$

$$= C_1 - \frac{\rho_3}{4\varepsilon_0}\left(R_2^2 - R_3^2\right) - \frac{\sigma R_2}{\varepsilon_0} \cdot \ln\left(\frac{R_2}{R_3}\right) - \left(\frac{\rho_1 R_1^2}{2\varepsilon_0} - \frac{\rho_3 R_2^2}{2\varepsilon_0}\right) \ln\left(\frac{R_2}{R_3}\right)$$

$$- \int_{R_2}^{r} \frac{\rho_1 R_1^2}{2\varepsilon_0} \cdot \frac{1}{r}\,dr$$

$$V(\vec{r}) = C_1 + C_2 - \frac{\rho_1 R_1^2}{2\varepsilon_0} \ln\left(\frac{r}{R_2}\right)$$

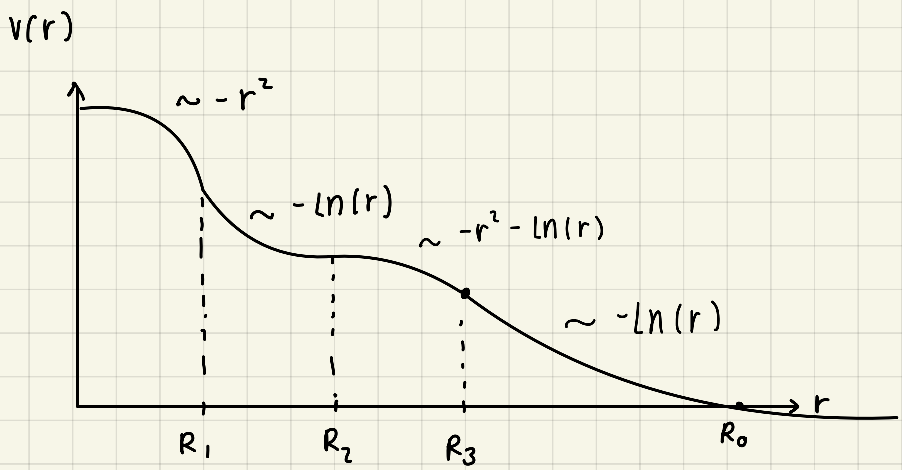
i.- $$V(\vec{r}) = -\underbrace{\int_{R_0}^{R_3} E(r > R_3)\,dr}_{C_1} - \underbrace{\int_{R_3}^{R_2} E(R_2 < r < R_3)\,dr}_{C_2} - \underbrace{\int_{R_2}^{R_1} E(R_1 < r < R_2)}_{C_3}$$

$$- \int_{R_1}^{r} E(0 < r < R_1)\,dr$$

$$= C_1 + C_2 - \frac{\rho_1 R_1^2}{2\varepsilon_0} \ln\left(\frac{R_1}{R_2}\right) - \int_{R_1}^{r} \frac{\rho_1 r}{2\varepsilon_0}\,dr$$

$$V(\vec{r}) = C_1 + C_2 + C_2 - \frac{\rho_1}{2\varepsilon_0}\left(\frac{r^2}{2} - \frac{R_1^2}{2}\right) \quad //$$

c)



* En R_0, $V(R_0) = 0$.