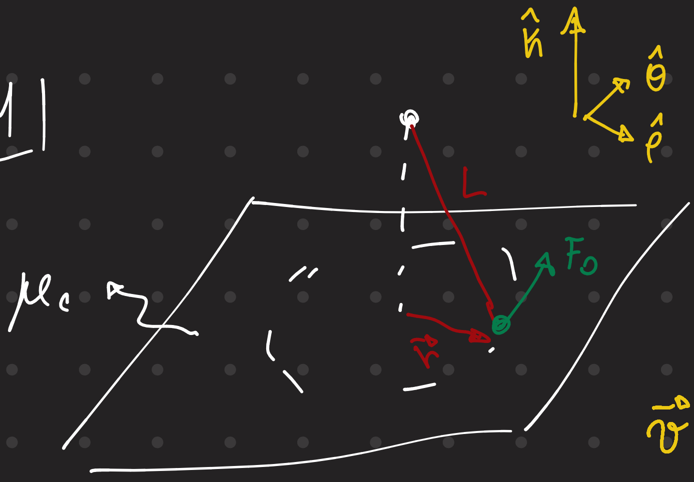
P1)



Cilíndricas:

$$\vec{r}(t) = \rho\,\hat{\rho} + z\,\hat{k}$$

$$\vec{v}(t) = \dot{\rho}\,\hat{\rho} + \rho\dot{\theta}\,\hat{\theta} + \dot{z}\,\hat{k}$$

$$\vec{a}(t) = (\ddot{\rho} - \rho\dot{\theta}^2)\,\hat{\rho} + (\rho\ddot{\theta} + 2\dot{\rho}\dot{\theta})\,\hat{\theta} + \ddot{z}\,\hat{k}$$

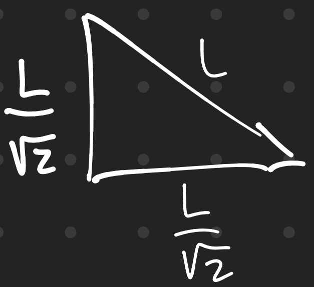
Cinemática

Se tiene que:

$$z(t) = 0 \Rightarrow \dot{z} = \ddot{z} = z = 0$$

Además:

$$\rho(t) = \frac{L}{\sqrt{2}} \Rightarrow \dot{\rho} = \ddot{\rho} = 0$$



Con esto:

$$\vec{r}(t) = \rho\,\hat{\rho} + \cancelto{0}{z\,\hat{k}}$$

$$\vec{v}(t) = \cancelto{0}{\dot{\rho}\,\hat{\rho}} + \rho\dot{\theta}\,\hat{\theta} + \cancelto{0}{\dot{z}\,\hat{k}}$$

$$\vec{a}(t) = (\cancelto{0}{\ddot{\rho}} - \rho\dot{\theta}^2)\,\hat{\rho} + (\rho\ddot{\theta} + \cancelto{0}{2\dot{\rho}\dot{\theta}})\,\hat{\theta} + \cancelto{0}{\ddot{z}\,\hat{k}}$$

$$\underbrace{\left\{\begin{array}{l} \vec{r}(t) = \frac{L}{\sqrt{2}}\,\hat{\rho} \\ \vec{v}(t) = \frac{L}{\sqrt{2}}\dot{\theta}\,\hat{\theta} \\ \vec{a}(t) = -\frac{L}{\sqrt{2}}\dot{\theta}^2\,\hat{\rho} + \frac{L}{\sqrt{2}}\ddot{\theta}\,\hat{\theta} \end{array}\right.}_{+1}$$

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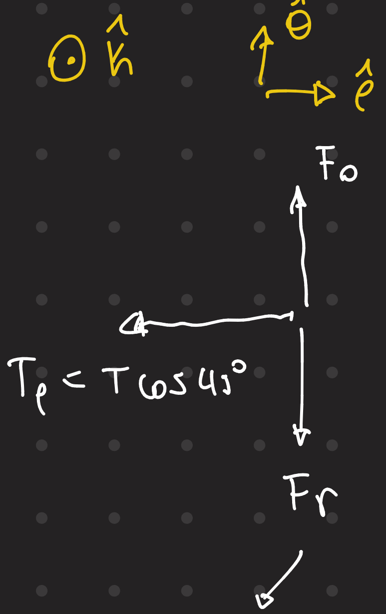
Dinámica

DCL m → +1



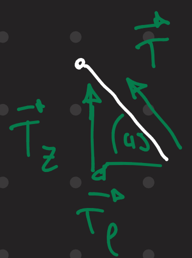
⊙ $T_z = T\operatorname{sen} 45°$

⊙ N

⊗ mg

F_r se opone al mov, y $\vec{v}$ solo tiene componente en $\hat{\theta}$

Respecto a la T:



$$\begin{cases} T_z = T\operatorname{sen} 45° \\ T_\rho = T\cos 45° \end{cases}$$

2° ley

$$\hat{\rho}\,|\; -T\cos 45° = m\left(-\frac{L}{\sqrt{2}}\right)\dot{\theta}^2$$

$$\hat{\theta}\,|\; F_o - F_r = m\frac{L}{\sqrt{2}}\ddot{\theta}$$

$$\Leftrightarrow F_o - \mu N = m\frac{L}{\sqrt{2}}\ddot{\theta}$$

$$\hat{z}\,|\; T\operatorname{sen} 45° + N - mg = 0$$

+1

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b) Buscamos $\dot{\theta}(\theta)$

$$\begin{cases} \cancel{-}T\dfrac{1}{\cancel{\sqrt{2}}} = \cancel{-}m\dfrac{L}{\cancel{\sqrt{2}}}\dot{\theta}^2 & (1) \\ F_0 - \mu N = m\dfrac{L}{\sqrt{2}}\ddot{\theta} & (2) \\ T\cdot\dfrac{1}{\sqrt{2}} + N - mg = 0 & (3) \end{cases}$$

En (1): $T = mL\dot{\theta}^2$

$\downarrow$

(3) $\dfrac{mL\dot{\theta}^2}{\sqrt{2}} + N - mg = 0$

$\Rightarrow N = mg - \dfrac{mL\dot{\theta}^2}{\sqrt{2}}$

$\downarrow$

(2) $F_0 - \mu mg + \dfrac{\mu m L\dot{\theta}^2}{\sqrt{2}} = m\dfrac{L}{\sqrt{2}}\ddot{\theta} \quad / \cdot \dfrac{\sqrt{2}}{mL}$

$$\Leftrightarrow \boxed{\frac{\sqrt{2}F_0}{mL} - \frac{\mu g\sqrt{2}}{L} + \mu\dot{\theta}^2 = \ddot{\theta}}$$

Ec. de Mov.

↳ +1

busco $\dfrac{d\dot{\theta}}{d\theta}$

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$$\frac{\sqrt{2}\,F_0}{mL} - \frac{\mu g \sqrt{2}}{L} + \mu \dot{\theta}^2 = \ddot{\theta}$$

$$\underbrace{\frac{\sqrt{2}\,F_0}{mL} - \frac{\mu g \sqrt{2}}{L}}_{\equiv\, K} + \mu \dot{\theta}^2 = \frac{d\dot{\theta}}{d\theta} \cdot \dot{\theta}$$

$$\Longleftrightarrow \quad K + \mu \dot{\theta}^2 = \frac{d\dot{\theta}}{d\theta} \cdot \dot{\theta}$$

$$\Longleftrightarrow \quad \left[K + \mu \dot{\theta}^2\right] d\theta = \dot{\theta}\, d\dot{\theta} \qquad / \int_0^{\theta}$$

$$\Longleftrightarrow \quad K\theta + \mu \int_0^{\theta} \dot{\theta}^2\, d\theta = \int \dot{\theta}\, d\dot{\theta}$$

$$\Longleftrightarrow \quad K\theta + \mu\, \theta \cdot \dot{\theta}^2 = \frac{\dot{\theta}^2}{2}$$

$$\Longleftrightarrow \quad K\theta = \frac{\dot{\theta}^2}{2} - \mu\, \theta \dot{\theta}^2$$

$$\Longleftrightarrow \quad K\theta = \dot{\theta}^2 \left[\frac{1}{2} - \mu\theta\right]$$

$$\Longleftrightarrow \quad \dot{\theta}^2 = \frac{K\theta}{\frac{1}{2} - \mu\theta} \quad \Longleftrightarrow \quad \dot{\theta} = \sqrt{\frac{K\theta}{\frac{1}{2} - \mu\theta}}$$

$$\dot{\theta} = \sqrt{\frac{k\theta}{\frac{1}{2} - \mu\theta}}$$

$+0.5$

$$\boxed{\dot{\theta}(\theta) = \sqrt{\frac{\left[\frac{\sqrt{2}F_0}{mL} - \frac{\mu g\sqrt{2}}{L}\right]\theta}{\frac{1}{2} - \mu\theta}}}$$

v. angul.

$$\frac{1}{s} \stackrel{?}{=} \sqrt{\frac{\frac{\cancel{kg}\,\frac{\cancel{m}}{s^2}}{\cancel{m}\cdot\cancel{kg}} - \frac{\frac{\cancel{m}}{s^2}}{\cancel{m}}}{1}} = \sqrt{\frac{1}{s^2}} = \frac{1}{s} \checkmark\checkmark$$

$$\underbrace{\hspace{6cm}}_{\text{Análisis dim} \checkmark}$$

↳ Estoy seguro q' se podía hacer + corto, pero son las 4 am y no pienso bien (pido perdón)

c) θ para que se despegue

↳ Imponemos cond. de despegue $(N \doteq 0)$ y despejamos θ

+1

$$\begin{cases} \cancel{-}T \dfrac{1}{\cancel{\sqrt{2}}} = \cancel{-} m \dfrac{L}{\cancel{\sqrt{2}}} \dot{\theta}^2 & (1) \\ F_0 - \cancel{\mu N}^{\,0} = m \dfrac{L}{\sqrt{2}} \ddot{\theta} & (2) \\ T \cdot \dfrac{1}{\sqrt{2}} + \cancel{N}^{\,0} - mg = 0 & (3) \end{cases}$$

$(3) \to \quad T = mg\sqrt{2}$

(1) ↳ $\cancel{m}g\sqrt{2} = \cancel{m}L\dot{\theta}^2 \Rightarrow \bar{\dot{\theta}} = \sqrt{\dfrac{g\sqrt{2}}{L}}$

↳ Cuándo $\dot{\theta} > \bar{\dot{\theta}}$ se despega

Sabemos que:

$\dot{\theta}(t) = \cancel{\dot{\theta}(0)}^{\,0} + \ddot{\theta}\, t$

$\ddot{\theta}$ es cte : $\ddot{\theta} = \dfrac{\sqrt{2}\,F_0}{Lm}$
(2)

luego :

$$\dot{\theta}(t) = \frac{\sqrt{2}\,F_0}{Lm}\cdot t$$

Impon. $\dot{\theta}(\bar{t}) = \bar{\dot{\theta}}$ obt. $\bar{t}$ de despegue

$$\Rightarrow \bar{t}\,\frac{\sqrt{2}\,F_0}{Lm} = \sqrt{\frac{g\sqrt{2}}{L}} \Rightarrow \bar{t} = \frac{Lm}{\sqrt{2}\,F_0}\sqrt{\frac{g\sqrt{2}}{L}}$$

Finalmente, por mcva:

$$\theta(t) = \cancel{\theta(0)}^{0} + \cancel{\dot{\theta}(0)}^{0}\,t + \frac{1}{2}\ddot{\theta}t^2$$

$$\Rightarrow \theta(t) = \frac{1}{2}\ddot{\theta}t^2 \Rightarrow \theta_s = \theta(\bar{t}) = \frac{1}{2}\ddot{\theta}\,\bar{t}^2$$

$$\Rightarrow \theta_s = \frac{1}{\cancel{2}}\frac{\cancel{\sqrt{2}}\,\cancel{F_0}}{\cancel{Lm}}\left(\frac{\cancel{L^2}\,m^{\cancel{2}}}{2F_0^{\cancel{2}}}\right)\cdot\frac{\cancel{\sqrt{2}}\,g}{\cancel{L}}$$

$$\Rightarrow \boxed{\theta_s = \frac{mg}{2F_0}}$$

Puntaje:

a) +3
+1 Cin. +1 DCL
+1 ecn.

b) +1.5
+1. trupeito
+.5 despeje

c) +1.5
+1 N≠0
+.5 despeje.