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Application of Mining Width-Constrained Open Pit Mine Production Scheduling Problem to the Medium-Term Planning of Radomiro Tomic Mine: A Case Study

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Abstract

This article presents a novel approach to address the mining width-constrained open pit mine production scheduling problem in the context of medium-term planning. A mathematical formulation is proposed to incorporate mining width constraints into the production scheduling process, aiming to maximize the NPV of the schedule while ensuring enough room for the operation of mining equipment. To tackle the computational challenges posed by large-scale instances of the problem, we propose a method based on variable fixing and horizontal precedence generation. In this study, we apply the developed model to real-world scenarios from Radomiro Tomic short-term mine planning problems such as optimizing the timing of major truck maintenance and the impact of external factors, like the delay in the production of the Chuquicamata underground project. Remarkable improvements are observed with the mining width-constrained model. Specifically, the mining width satisfiability is enhanced from 2 to 60% compared to the traditional open pit mine production scheduling model, underscoring the significance of incorporating these constraints. The proposed method showed good results reaching optimality gaps within 5%.

Keywords Open pit · Mine planning · Mining width · Direct block scheduling · OPMPS

1 Introduction

The effective management of mining operations relies on optimizing the resource utilization and production schedule. In this context, the application of scheduling models has emerged as a promising approach to enhance short- and medium-term mine planning strategies. Radomiro Tomic mine (RT), operated by Codelco Chile, stands as a significant copper mining operation renowned for its rich mineral deposits and strategic importance in the global copper industry. Located 40 km from the city of Calama in the Antofagasta region of Chile, the mine operates at an elevation of 3000 m above sea level. While its operations officially commenced in 1995, the discovery of RT dates to the 1950s. Through continuous technological advancements and feasibility studies, Codelco successfully established the mine as an economically viable operation, leveraging its vast reserves of oxidized and sulfide minerals. As one of the largest copper mine worldwide, RT plays a crucial role in meeting global copper demand and contributing to the economic growth of Chile.

Highlights

- New formulation that integrates mining width constraints into open-pit production scheduling.
- Adaptation of algorithms and heuristics to manage large-scale instances.
- Scenario analysis based on Radomiro Tomic real dataset offers practical insights for medium-term mine planning optimization.
- The new formulation boosts mining width satisfiability to 60% compared to the 2% obtained with traditional models, with minimal optimality gaps.

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The geology of RT is primarily associated with the Chuquicamata porphyry complex, a copper-rich geological formation. The mine houses a porphyry copper deposit formed by molten magma intrusion, resulting in mineralized zones with copper and other valuable elements. These zones include the leached, oxide, mixed, enrichment, and primary sulfide zones, each with distinct characteristics. RT employs open-pit mining to extract oxidized minerals from the leached and oxide zones, using crushing, leaching, and solvent extraction-electrowinning techniques. For sulfide minerals in the mixed, enrichment, and primary sulfide zones, more complex methods like crushing, grinding, and froth flotation are required. Open-pit mining allows for efficient extraction due to its large-scale operation and specialized equipment and infrastructure.

Mine planning in RT is commonly done in four stages. Strategic planning includes the company's vision, looking for alternatives to create more value for the company. Once an alternative is chosen, it is developed into the business plan, which considers the Life of Mine (LOM). The medium-term plan shows more detail in the first 5 years of production. The main scope is to ensure the conditions for fulfilling the business plan. Short-term plans are typically focused on the first year of the medium-term plan, done on a monthly scale. Periods shorter than that are known as operational plans (monthly, weekly, or daily plans).

Medium- and short-term mine planning holds immense significance in optimizing production and resource utilization. It serves as a vital link between the long-term strategic plans and the day-to-day operational activities of a mine. By focusing on shorter time horizons, typically ranging from weeks to months, short-term mine planning allows mining companies to adapt to changing market conditions, optimize production rates, and maximize the efficient utilization of resources. This planning phase involves intricate considerations such as determining the optimal sequence of mining activities [1, 2], coordinating equipment allocation (e.g., [3–6]), and managing the extraction of various ore types and grades [7–9]. By aligning these factors, it is ensured the continuous flow of ore to the processing plant, minimizes ore loss and dilution, and enhances overall productivity. It enables mining companies to achieve production targets, meet quality specifications, and maintain a steady supply of minerals to the market, contributing to their profitability and sustainable growth.

Medium- and short-term mine planning involves a myriad of challenges and complexities that need to be carefully addressed [10]. One such challenge is managing multiple mining phases (or pushbacks) within the same operational period [11, 12]. In open-pit mining, for example, it is common to have concurrent operations in different areas of the mine, including pre-stripping, waste removal, and ore extraction. Coordinating these activities while optimizing equipment utilization and maintaining safety standards requires careful planning and scheduling. Additionally, short-term mine planning must consider processing alternatives to accommodate variations in ore characteristics. This entails evaluating different processing routes, such as crushing, grinding, and flotation, to achieve the desired product specifications efficiently [13, 14]. The coordination of mining and processing activities is crucial to avoid bottlenecks and to ensure a smooth flow of material through the production chain. Managing these complexities demands a comprehensive understanding of the mining operation, the interdependencies between different activities, and the ability to make informed decisions in real-time.

The challenges associated with short-term mine planning necessitate the adoption of advanced planning models and techniques. Traditional manual planning approaches are often time-consuming, subject to human errors, and limited in their ability to consider multiple constraints and objectives simultaneously [12, 15]. To address these limitations, the mining industry has witnessed the emergence of advanced planning models that leverage technologies such as optimization algorithms, mathematical programming, and simulation. These models enable mining companies to incorporate various factors, including geological uncertainties, equipment capacities, processing constraints, and market dynamics, into the planning process [16]. By utilizing real-time data, these models can generate optimal short-term production schedules, considering the dynamic nature of mining operations [17, 18].

Open-pit mine production scheduling models have emerged as a valuable tool in mine planning, offering a systematic approach to optimize the extraction sequence. These models consider the discretization of the mining area into blocks, considering factors such as geological characteristics, mining constraints, and production targets. By incorporating various parameters and constraints, block scheduling models enable planners to define the best extraction period and process destination for each block.

Bienstock and Zuckerberg [19] introduce an algorithm aimed at resolving the linear programming relaxation related to the precedence-constrained production scheduling problem (PCPSP). The innovation lies in a reformulation strategy that clusters numerous variables into a singular representation. The problem is decomposed into a dual (P2) and a primal (P1) problem. The primal problem is the relaxation of the PCPSP (removing the side constraints). Thus, the subproblem P1 can be efficiently solved using maximal closure algorithms. The information from the P1 is used to cluster the variables that are considered in the P2 problem, which has the PCPSP structure and due to the significant variable reduction can be efficiently solved to optimality. The dual values associated to the side constraints of the solution of the P2 are then considered for the P1 (like a Lagrangian methodology). In broad terms, the algorithm iterates until the clusters remain the same. Comprehensive studies on Bienstock-Zuckerberg algorithm applied to the Open Pit Mine Production Scheduling (OPMPS) can be found in Letelier et al. [20] and Muñoz et al. [21].

Chicoisne et al. [22] present the Critical Multiplier Algorithm targeted at addressing linear programming relaxations of large instances of the PCPSP when only one side constraint is considered. Their method consists of building a LP solution based on a systematic penalization of the profits used to compute an ultimate pit (as in [23]). Thus, the *nested pits* obtained from the penalization method are used to build the solution of the LP relaxation for the single resource constrained PCPSP (also referred as C-PIT). They also introduce a suite of heuristics based on topological sorting to build integer solutions to the C-PIT problem.

However, traditional OPMPS models often yield non-mineable solutions due to the scarce distribution of blocks mined at the same periods. Therefore, mine practitioners, in most cases, must significantly modify the solution to the implementation in the mine planning process. Thus, the inclusion of minimum mining width conditions in OPMPS models may enhance the mineability of medium-term plans, reducing the portion of the solution that must be modified before its implementation.

Recent work on the inclusion of mining width constraints in similar open pit problems, like the ultimate pit limit, nested pits, and pushbacks design problems, has resulted in substantial improvements on the mineability and practicality of the solutions. For example, Deutsch [24] introduces techniques based on Boolean satisfiability for production planning with mining width constraints, though not immediately applicable to large-scale problems. Deutsch et al. [25] present a dynamic programming algorithm for evenly spaced pushbacks, addressing the *gap* issue (uneven variation of volume between pits) in the resultant design. Nancel-Penard and Morales [26] expand an integer linear programming model for pit design with added mining width constraints, achieving smoother operational designs. Yarmuch et al. [27] focus on mining width and continuity in pushback generation using rectangular templates and flow constraints, while Yarmuch and Rubinstein [28] propose a soap-bubble cluster method for efficient pushback shapes. Morales et al. [29] develop an integer programming model for nested pits with penalty-based constraints, impacting resulting geometries and economic value. Transitioning to production scheduling, a recent work by Nancel-Penard and Jelvez [30] proposes an integer linear programming model with a decomposition heuristic for mining width requirements, ensuring safe pit walls and operational constraints, though limited by instance size. However, most of these approaches are neither developed for the OPMPS problem nor capable of dealing with large instance problems. Therefore, this study will focus on developing a new framework to include such constraints to the OPMPS and solve real world problem instances.

Specifically, the objective of this paper is to present a mathematical model that includes mining width constraints to the OPMPS problem (MW-OPMPS) and solve it in the context of medium-term mine-planning. We utilize the Bienstock-Zuckerberg LP (BZ) algorithm to solve subproblems associated with the MW-OPMPS. Additionally, the paper seeks to adapt the method introduced in Yarmuch et al. [27] to account for the mining width constraint. The MW-OPMPS model offers a more suitable framework for this highly complex mining operation. The use of BZ algorithm is particularly relevant at RT, where materials can be processed in different processing plants, making the application of alternative methods such as CMA less effective.

The subsequent sections of this paper provide a comprehensive analysis of the application of a MW-OPMPS model at RT, focusing on optimizing medium-term mine planning processes. In Sect. 2, we present a mathematical formulation for the MW-OPMPS and describe our solution method. Section 3 presents a detailed case study aimed at answering multiple operational issues providing valuable insights into the practical use of the MW-OPMPS model at a prominent copper mining operation. Finally, in Sect. 4, we present the conclusions drawn from the study.

2 Mathematical Model and Solution Method

In this section, we introduce a mathematical model aimed to maximize the net present value (NPV) of an open pit mine's production schedule while adhering to processing and geometrical constraints. It considers processing and mining upper limits per period, geotechnical wall slope constraints modeled as a set of block precedence, and multiple destinations for the mined blocks, such as heap leach, mills, and waste dumps. To ensure its realism and applicability in medium-term planning, the model accounts for the mining width constraint per period—for simplicity—modeled as a series of rectangular templates (see [27, 31]) and a maximum vertical advance (also known as sinking rate) modeled as a set of inter-temporal set of precedencies. The sinking rate constraints are important to prevent a deep isolated excavation at a given period mimicking more bench-by-bench excavation.

2.1 Sets and Notation

$b \in B$: set of blocks.

$t \in \mathcal{T}$: periods (a month or a few months in this case).

$d \in \mathcal{D}$: destinations.

$w \in \mathcal{W}$: set of all rectangular templates.

$\hat{w} \in \mathcal{W}_b$: set of rectangular templates that contains block b

$\widehat{b} \in \mathcal{B}_w$: set of blocks that are contained within the template w

$(u, v) \in \mathcal{S}$: set of pair of blocks that are in conflict regarding the sinking rate constraint, i.e., block v needs to be extracted at least a period earlier than block u

$(a, b) \in \mathcal{A}$: set of pair of blocks that forms the geotechnical constraints, i.e., block b needs to be extracted before than block a

2.2 Parameters

p^*_{bdt}: discounted profit of block b when extracted at period t and sent to destination d. Computed as $p^*_{bdt} = \frac{p_{bd}}{(1+\rho)^t}$, where p_{bd} is the profit of block b when sent to destination d and ρ is the discount rate.

$a_{b,d}$: resource associated to block b at destination d

$C_{d,t}$: maximum resource capacity (tonnage) of destination d at period t

2.3 Variables

The first set of variables are associated with the extraction of the blocks.

$$x_{b,d,t} = \begin{cases} 1 & \text{if block } b \text{ is sent to destination } d \text{ at period } t \\ 0, & \textit{otherwise} \end{cases}$$

The second set of variables are associated to the ramp segments considered to build the ramp.

$$u_{w,t} = \begin{cases} 1, & \text{if the mining width template } w \text{ is selected at period } t \\ 0, & \textit{otherwise} \end{cases}$$

2.4 Objective Function

$$\max \sum_{b\in\mathcal{B}} \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} p^*_{b,d,t} \cdot x_{b,d,t}$$

The objective function aims to maximize the discounted cashflow or net present value.

2.5 Constraints

$$\sum_{b\in\mathcal{B}} a_{b,d} \cdot x_{b,d,t} \le C_{d,t} \quad t \in \mathcal{T}, d \in \mathcal{D} \tag{1}$$

$$\sum_{\tau\in\{1,..,t\}} \sum_{d\in D} x_{a,d,\tau} \le \sum_{\tau\in\{1,..,t\}} \sum_{d\in D} x_{b,d,\tau} \quad (a, b) \in \mathcal{A} \tag{2}$$

$$\sum_{d\in\mathcal{D}} x_{b,d,t} \le \sum_{\widehat{w}\in\mathcal{W}_b} u_{\widehat{w},t} \quad b \in \mathcal{B}, \mathrm{t} \in \mathcal{T} b \in \mathcal{B}, \mathrm{t} \in \mathcal{T} \tag{3}$$

$$u_{w,t} \le \sum_{d\in\mathcal{D}} x_{b,d,t} \quad w \in \mathcal{W}, \widehat{b} \in \mathcal{B}_w, \mathrm{t} \in \mathcal{T} \tag{4}$$

$$\sum_{t\in\mathcal{T}} \sum_{d\in\mathcal{D}} x_{b,d,t} \le 1 b \in \mathcal{B} \tag{5}$$

$$\sum_{t\in\mathcal{T}} u_{w,t} \le 1 \quad w \in \mathcal{W} \tag{6}$$

$$\sum_{\tau\in\{1,..,t\}} \sum_{d\in\mathcal{D}} x_{i,d,\tau} \le \sum_{\tau\in\{1,..,t-1\}} \sum_{d\in\mathcal{D}} x_{j,d,\tau} \quad (i, j) \in \mathcal{S} \tag{7}$$

Constraint (1) represents the resource constraint per destination per period, in our case, the resource associated with each block consists of its tonnage. This constraint can directly be extended to multiple resource constraints per destination. Constraint (2) ensures that, to extract a block b at period t, all its vertical predecessors must also be extracted by period t. Constraint (3) flags a rectangular template w that contains b if block b is mined at period t. Constraint (4) forces all blocks within block template w to be extracted at period t. Constraint (5) ensures that a block is extracted and sent at most to one destination over the periods. Constraint (6) ensures that the rectangular templates are selected on at most one period. Finally, Constraint (7) ensures that all the blocks that could violate the sinking rate constraint from block i are extracted at least a period earlier than block j itself.

2.6 Solution Method

The resultant MW-OPMPS integer programming model, hereafter named as P, is intractable for large problem instances. Therefore, we propose to solve a slightly relaxed version of P denoted as P*, that captures most of the structure of P. The solution method is as follows. First, let Q denote as the subproblem formed by Constraints (1), (2), (5), and (7). Second, we define the problem P* as the sub-problem Q plus the following constraint: $\sum_{\tau\in\{1,...,t\}} \sum_{d\in\mathcal{D}} x_{l,d,\tau} \le \sum_{\tau\in\{1,...,t\}} \sum_{d\in\mathcal{D}} x_{m,d,\tau}, (l, m) \in \mathcal{H}$(Constraint (8)), i.e., we replace Constraints (3), (4) and (6) by Constraint (8) (Fig. 1).

The set of horizontal precedencies $\mathcal{H}$ is obtained by adapting the procedure introduced in Yarmuch et al. (2021b), as follows. First, we solve the subproblem Q which its LP relaxation is solved with the BZ algorithm. The integer solution is, then, obtained using the *TopoHeur* procedure presented in Chicoisne et al. [22]. Second, the predefined rectangular template is floated over the solution of Q.

As a block can be included in multiple templates, the procedure consists of selecting the templates that present the smaller average extraction time for each block and then creating the set of horizontal precedencies $\mathcal{H}$ as shown in

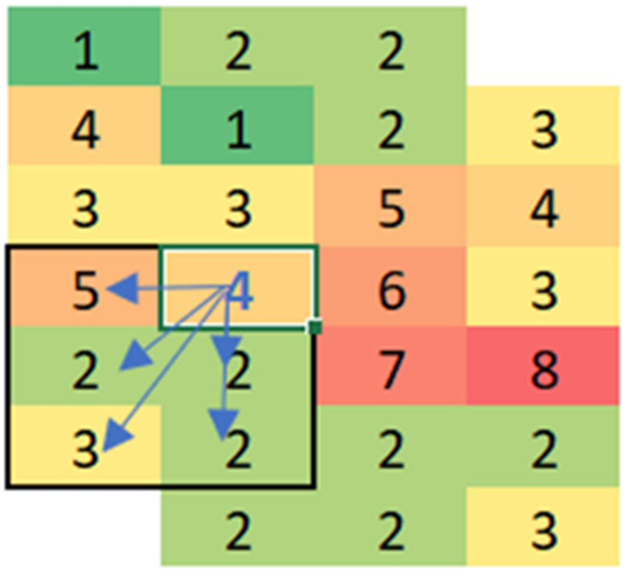


Fig. 1 Illustration of the creation of horizontal precedence (blue arrows) based on a modification of the template variable fixing introduced in Yarmuch et al. (2021b)

Fig. 1. In the example, a rectangular template of 2 by 3 blocks is considered. Fig. 2 illustrates the average period corresponding to each template that contains the selected block in Fig. 1. Thus, the linear relaxation of P* preserves the mathematical structure of the OPMPS, and it can be solved using the same method described to solve the sub-problem Q, but now considering the precedencies $\mathcal{H}$ for the TopoSort and TopoHeur rounding heuristic.

However, there are two main limitations of this procedure. First, the method of choosing the minimum average template does not ensure that all the blocks within a template will select the same template for the creation of the set $\mathcal{H}$. Therefore, we developed some rules for the addition of the precedencies to the set $\mathcal{H}$, following a first come first serve strategy. For example, the precedence (l, m) is added only if the precedence (m, l) is not in $\mathcal{H}$; also, we give priority to ore blocks.

Second, we solve the problem P^* using the -by formulation to exploit the network structure of the problem (see Appendix). In that regard, the precedencies $\mathcal{H}$ will be imposed as the block m must be extracted by the time the block l is extracted. This is not what we are modeling in P, where the relationship is that the blocks within the same template must be extracted at the same period, not by the same period.

As this procedure precludes, the full compliance of the mining width in the solution of P^*, we define a simple metric named mining width satisfiability (MWS), to measure the mining width compliance. The metric basically counts the number of blocks (N_w) for which exists a template that (1) contains a block, and all the blocks within that template are extracted at the same period. That number is then divided by the total of blocks scheduled (N_s). Thus, the mining width satisfiability is obtained as the division of N_w and N_s ($MWS = N_w/N_s$).

3 Case Study

In this case study, we conducted a set of four experiments to investigate the incorporation of mining width constraints in the OPMPS problem and its applications to real medium-term scheduling problems. Experiment 1 considers the capacities from the baseline case but introduces operational mining widths of a 4×4 blocks template. The objective is

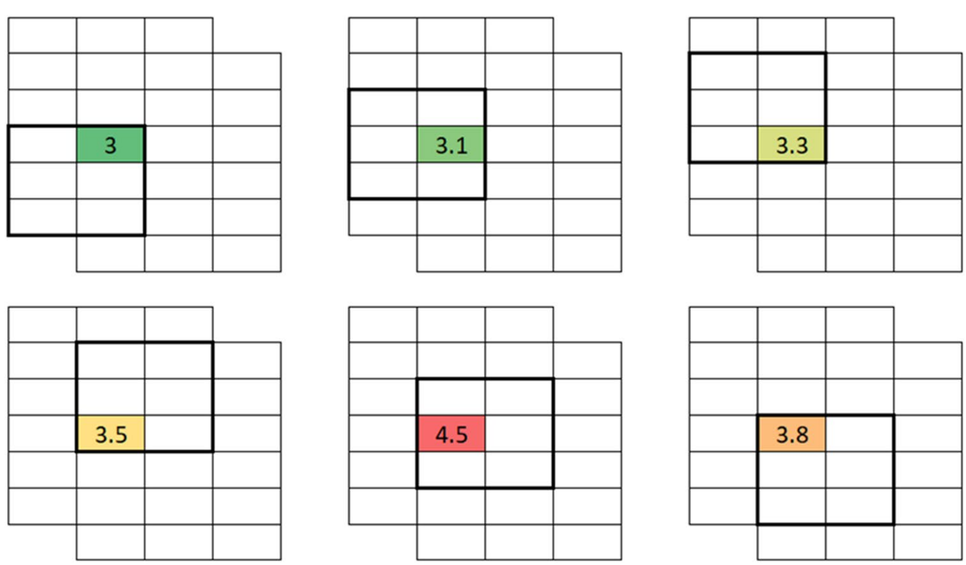


Fig. 2 Example of the average extraction period obtained for the rectangular templates that contains the colored block based on the solution of the sub-problem Q. The template with the smaller average is selected for the construction of the horizontal precedencies for the problem P*, as in Fig. 1

to compare a traditional OPMPS solution with the MW-OPMPS solution. We expect that the traditional OPMPS plan will appear less mineable, i.e., scarce spatial distribution of blocks mined at the same periods. The key focus is not only on understanding the improvement in mining width satisfiability but also assessing the impact on the NPV and the change in sequence compared to the traditional OPMPS solution.

Experiments 2, 3, and 4 encompass different operational scenarios. Experiment 2 evaluates the impact on the mining sequence, production schedule, and NPV of adjusting the total mining capacity to match it to real performance. Experiment 3 helps to estimate the impact and the best period for a major truck maintenance. Experiment 4 re-evaluates the impact of the major maintenance decision studied in Experiment 3 but considering an extra requirement of sulfide ore due to a delay of Chuquicamata underground mine (Chuqui UG).

We ran 10 problem instances to conduct the four experiments. Table 1 summarizes the instances parameters in relationship with the experiments conducted. Instances I1 and I2 consider the same capacities and parameters of the long-term plan (LOM), differing only on the addition of a mining width template of 4×4 for I2. Experiment 2 compares results of instance I3 against instance I2, to evaluate the impact of adjusting planned production to real production data. Instance I3 considers a 20% penalization in the LOM capacities due to truck availability issues. Instances I4, I5, and I6 not only consider the same mining capacity as instance I3 but also include a reduction of 35% of the LOM capacity in periods 2, 4, and 6 (months 3–4, 7–8, and 11–12), respectively. Subsequent periods after the maintenance period consider a 10% penalization compared to the LOM capacity.

Experiment 3 compares the results of instances I4, I5, and I6 against instance I3. For UG delay, we consider the truck penalization of instance I3 plus an extra requirement of 60 k tons per day of sulfide ore for months 13 to 24 (instance I7). Instances I8, I9, and I10 are based on instance I8 plus the maintenance scenarios of instances I4, I5, and I6, respectively. Finally, Experiment 4 consists of comparing instances I8, I9, and I10 against instance I7. The experiments described aim to evaluate the usefulness of the MW-OPMPS model and solution method to investigate the effects of varying operational conditions and provide insights into optimizing the scheduling process in complex mining environments.

The RT geological block model consists of 6.3 million blocks of size 20×20×15 m each. However, we only consider the blocks within the pushbacks planned for the first 5 years of the LOM, resulting in 47 thousand blocks to impose the use blocks within the LOM designed pushbacks. All experiments are run on a Windows machine with an 8th generation Intel i7 processor and 64 Gb RAM; we use the MILP software library CPLEX 22.1 to solve the LP problems. All mine schedules are run for the first 4 years of the operation (discretized as 24 bimonthly periods). A maximum sinking rate of 4 benches per period is considered for all the instances studied to control the vertical advance. The value of the LP relaxation of the OPMPS is used as the upper bound to measure the gap of the integer solution. Similarly, we use the value of the LP relaxation of the problem P^* as the upper bound for the MW-OPMPS instances.

3.1 Inclusion of the Mining Width Constraints

Experiment 1 shows the impact of including the mining width constraints on the OPMPS problem by solving the MW-OPMPS model introduced in Sect. 2. The findings revealed notable differences when comparing the mining width satisfiability of the solutions of the MW-OPMPS against the OPMPS. The same production, mining, and sinking rate constraints are considered for both cases. However, the MW-OPMPS considers a mining width template of 4 by 4 blocks. The same template is used to compute the mining width satisfiability.

As a result of Experiment 1, the solution of the OPMPS problem took a computation time of 18 min, while the solution of the MW-OPMPS problem took 37 min to solve. In terms of mining width satisfiability, the OPMPS achieved a very low satisfaction rate of 2.47%, while the satisfiability of the MW-OPMPS solution reached a significantly higher mining width compliance of 57% (see Fig. 3 and Fig. 4 for visual comparison).

Table 1 Parameters and description of problem instances studied for the experiments. *Inst.*, instance; *M.W.*, mining width templates (number of blocks in *X* direction and number of blocks in *Y* direction); *Prod. Adj.*, production adjustments; *Maint. Period.*, maintenance period; *UG delay*, the underground delay consideration

Inst	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10
M.W	1×1	4×4	4×4	4×4	4×4	4×4	4×4	4×4	4×4	4×4
Prod. Adj	-	-	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Maint. Period. [months]	-	-	-	3–4	7–8	11–12	-	3–4	7–8	11–12
UG delay	-	-	-	-	-	-	Yes	Yes	Yes	Yes

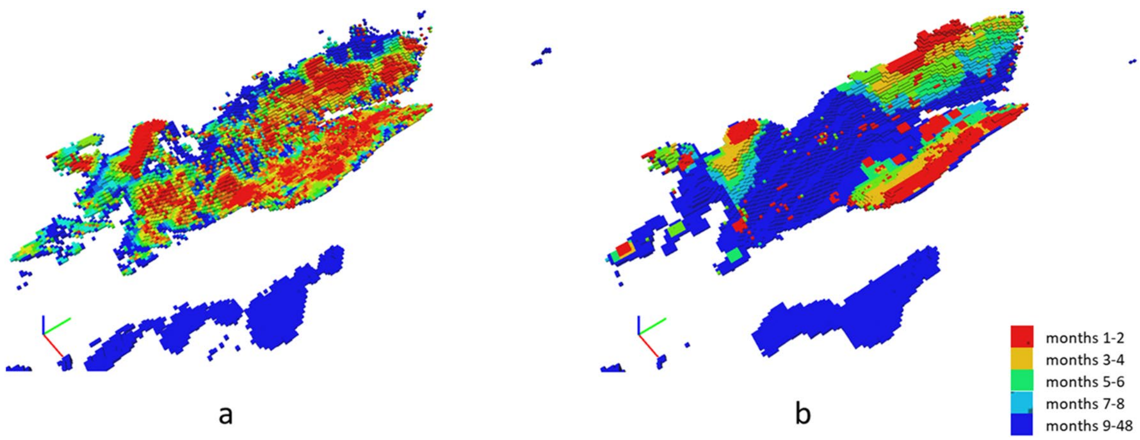
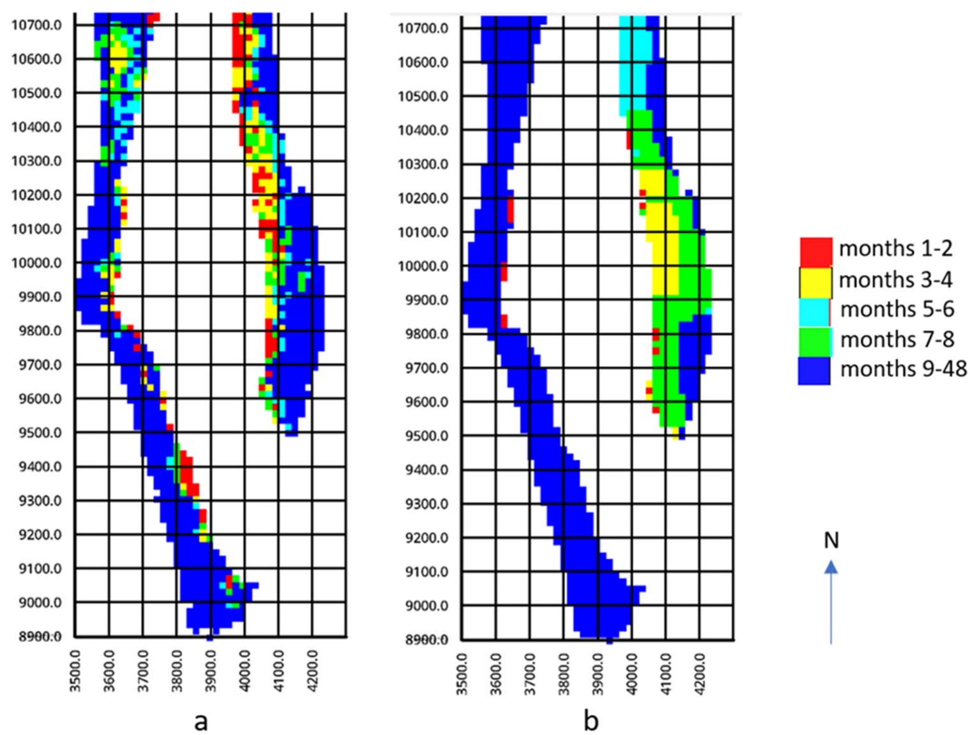


Fig. 3 Isometric view of Experiment 1 solutions, **a** solution of instance I1 and **b** the solution of instance I2

Fig. 4 Plan view (level 2610) of Experiment 1 results, **a** solution of instance I1 and **b** solution of instance I2



Remarkably, the inclusion of mining width constraints had a minor impact on the NPV of the mine plan, compared to the significant improvement in mineability. The OPMPS solution yielded an NPV of 1993 MUSD, while the MW-OPMPS solution resulted in a slightly lower NPV of 1862 MUSD. The optimality gap, representing the deviation from the upper bound considered as the Linear Relaxation integer models, was 0.1% for the OPMPS and 2.57% for the MW-OPMPS solutions. These gaps indicate that both approaches were relatively close to the optimal solution. Table 2 summarizes the results of Experiment 1.

The results of Experiment 1 highlight the effectiveness of incorporating mining width constraints in ensuring a greater proportion of blocks satisfies the width requirements, leading to improved operational compliance, and making the outputs more useful for engineering designs.

Furthermore, when considering the operational aspects of the mine plan, additional insights emerge from the inclusion of mining width constraints in the production scheduling problem. One significant observation is that the MW-OPMPS problem necessitates a significantly larger movement of waste compared to the OPMPS option. This

Table 2 Summary results of Experiment 1. *Inst.*, instance; *M.W.*, mining width template; *Maint. Period.*, maintenance period; *M.W. Comp.*, mining width compliance; *U.B.*, upper bound; *Obj. Val.*, the objective value; *Gap*, the optimality gap; and *Time*, the running time

Inst	M.W	Prod. Adj	Maint. Period. [months]	UG delay	M.W. Comp. [%]	U.B. [MUS$]	Obj. Val. [MUS$]	Gap [%]	Time [s]
I1	1×1	-	-	-	2.47	1995	1993	0.10	1096
I2	4×4	-	-	-	56.87	1910	1862	2.58	2214

implies that the inclusion of mining width constraints increases the amount of waste material that needs to be handled. Despite this, it is noteworthy that the MW-OPMPS effectively keeps the Run-of-Mine (ROM) and Mill to their full capacity, ensuring optimal throughput and maintaining production levels. Moreover, the analysis of NPV flow highlights that the NPV generated from the Mill plays a crucial role in supporting the overall NPV. Although the inclusion of mining width constraints slightly reduces the total NPV compared to the unconstrained problem, the majority of the NPV flow is sustained by the economic value generated from the Mill (see Fig. 5).

3.2 Scenario Analysis

3.2.1 Impact of Truck Availability

Experiment 2 aims to analyze the impact of truck availability on the mine schedule. In this scenario, the mine's production capacity is adjusted to model non-planned underperformance due to mechanical availability issues.

A comparison between the optimized base case and the adjusted production plan reveals notable differences. The NPV experiences a slight decrease from 1862 to 1802 MUSD. The adjusted production plan, that considers 20% less mining capacity than the LOM, involves delaying production on the northwest side of the mine and reducing stripping during the initial periods. The change on the mining sequence, and therefore the reduction, of stripping is a consequence of prioritizing valuable materials for the Heap Leach and the Mill at expenses of low-grade oxidized material for ROM.

Moreover, the NPV flow analysis shows that the adjusted case relies more on the Mill for generating NPV, while slightly reducing the NPV flow from the ROM and Heap leach (Fig. 6). The optimality gap remains relatively similar at 2.57% for the base case and 2.38% for the adjusted case. Table 3 summarizes the results from Experiment 2.

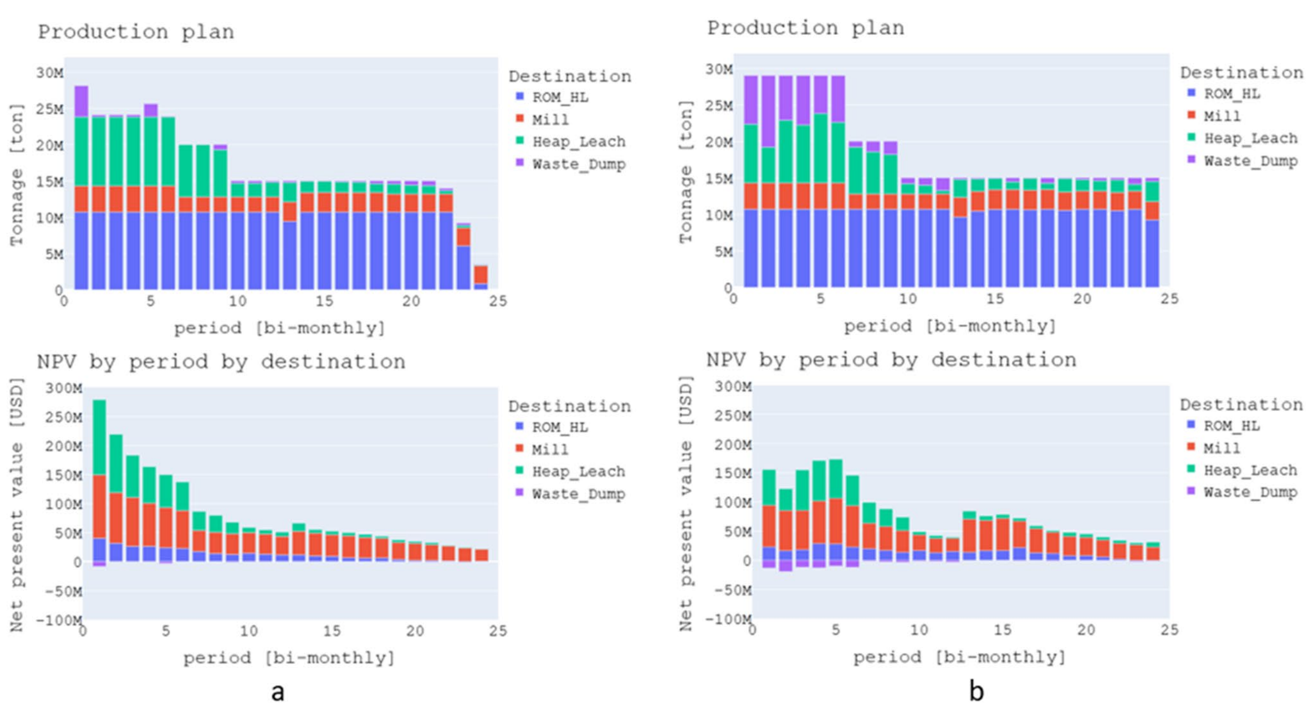


Fig. 5 Production plan (above) and NPV by period and destination (below) for the OPMPS solution (**a**) and the MW-OPMSP solution (**b**), Experiment 2

3.2.2 Best Period for Major Truck Maintenance

Experiment 3 aims to determine the impact of different periods for major truck maintenance, with three different periods (M1, M2, and M3) evaluated. The production-adjusted case is considered the base case to compare the major truck maintenance options (Table 4). Each maintenance period involves a 35% reduction in production, followed by a 10% recovery penalty in subsequent periods. Periods M1, M2, and M3 are period 2, period 4, and period 6, respectively.

Mining width satisfiability for the M1, M2, and M3 cases range from 61.89 to 62.91%, with an optimality gap ranging from 1.79 to 2.97%, making the solutions comparable between the cases. The NPV values for the three maintenance scenarios range from 1816 to 1842 MUSD, with being M1 the best alternative.

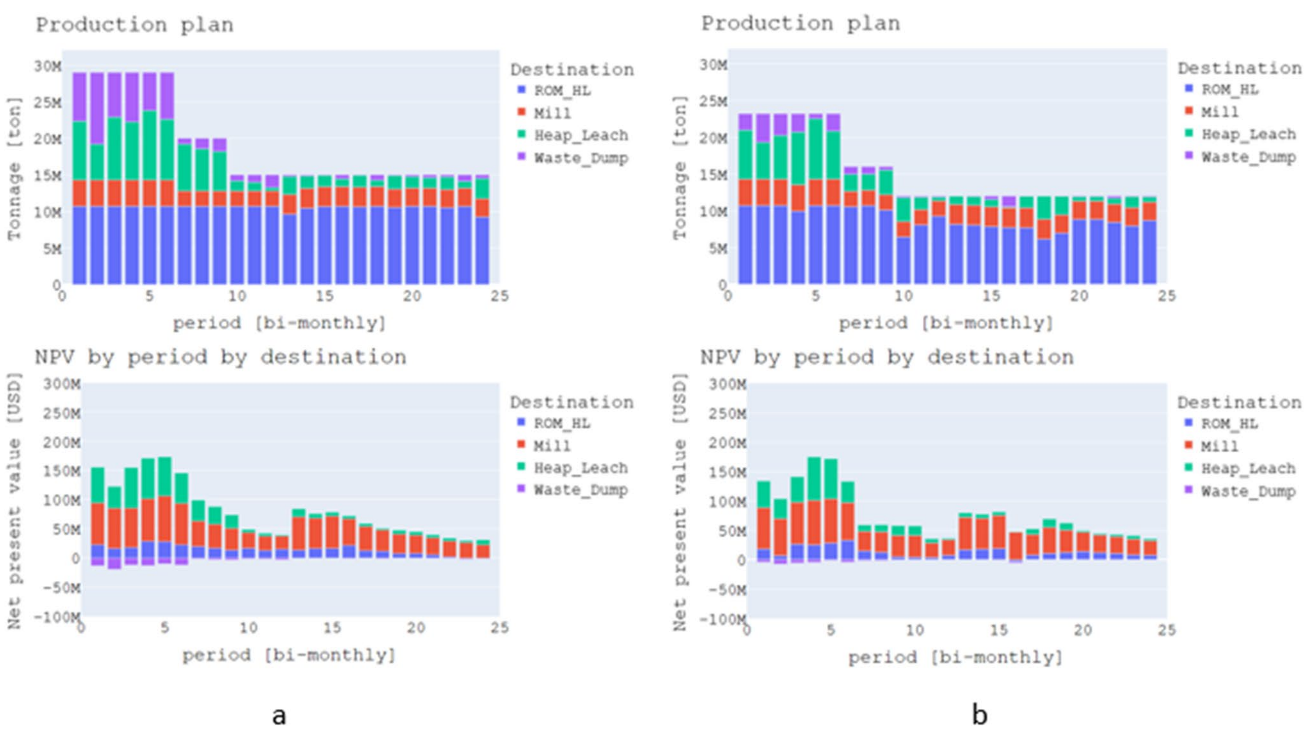


Fig. 6 Production plan (above) and NPV by period and destination (below) for the base case (**a**) and the production adjusted case (**b**)

Table 3 Summary results of Experiment 2. *Inst.*, instance; *M.W.*, mining width template; *Maint. Period.*, maintenance period; *M.W. Comp.*, mining width compliance; *U.B.*, upper bound; *Obj. Val.*, the objective value; *Gap*, the optimality gap; *Time*, the running time

Inst	M.W	Prod. Adj	Maint. Period. [months]	UG delay	M.W. Comp. [%]	U.B. [MUS$]	Obj. Val. [MUS$]	Gap [%]	Time [s]
I2	4×4	-	-	-	56.87	1910	1862	2.58	2214
I3	4×4	Yes	-	-	65.38	1845	1802	2.39	2235

Table 4 Summary results of Experiment 3. *Inst.*, instance; *M.W.*, mining width template; *Maint. Period.*, maintenance period; *M.W. Comp.*, mining width compliance; *U.B.*, upper bound; *Obj. Val.*, the objective value; *Gap*, the optimality gap; *Time*, the running time

Inst	M.W	Prod. Adj	Maint. Period. [months]	UG delay	M.W. Comp. [%]	U.B. [MUS$]	Obj. Val. [MUS$]	Gap [%]	Time [s]
I4	4×4	Yes	3–4	-	61.89	1875	1842	1.79	2161
I5	4×4	Yes	7–8	-	62.28	1870	1816	2.97	2516
I6	4×4	Yes	11–12	-	62.91	1865	1820	2.47	2448

The mining sequence analysis reveals variations in production timing in different regions of the mine, with delays observed in the southwest area for M1 and M3, while M2 brings forward material from the northwest side (see Fig. 7). All production plans are reasonably smooth and practical for being a raw output from a direct block schedule.

3.2.3 Impact of UG Delay

The final scenario, Experiment 4, considers a year delay in production from a neighboring underground mine, requiring a throughput increase of 60 ktpd from RT. The comparison between the production-adjusted case (Instance I3 of Table 3) and the production-adjusted + UG case (Instance I8) shows marginal changes in mining width satisfiability (64.44 to 65.38%) and NPV (1716 to 1802 MUSD). The optimality gap obtained was small for both cases, 2.38% and 3.20% respectively.

No major differences in the mining sequences are observed. Additionally, three maintenance scenarios (M1UG, M2UG, and M3UG) are evaluated, exhibiting mining width satisfiability ranging from 60.88 to 64.42% and optimality gaps from 2.84 to 3.12% (see Table 5).

The NPV for these scenarios' ranges from 1759 to 1769 MUSD, being scenario M3UG the best alternative for these conditions. The mining sequence analysis reveals delays on the northwest side and accelerated production on the southwest side for the M1UG, M2UG, and M3UG cases (see Fig. 8).

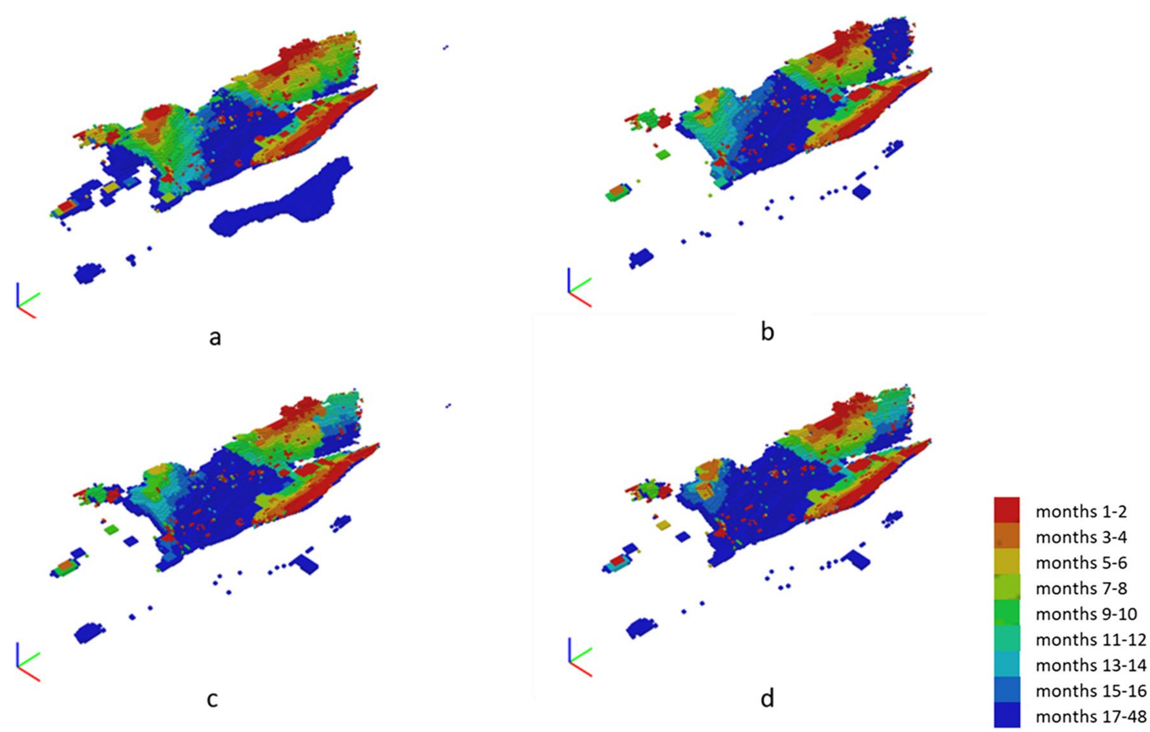


Fig. 7 Illustration of the spatial distribution of the first 18 months of production. **a** Production adjusted base case, **b** maintenance scenario M1, **c** maintenance scenario M2, and **d** maintenance scenario M3

Table 5 Summary results of Experiment 4. *Inst.*, instance; *M.W.*, mining width template; *Maint. Period.*, maintenance period; *M.W. Comp.*, mining width compliance; *U.B.*, upper bound; *Obj. Val.*, the objective value; *Gap*, the optimality gap; *Time*, the running time

Inst	M.W	Prod. Adj	Maint. Period. [months]	UG delay	M.W. Comp. [%]	U.B. [MUS$]	Obj. Val. [MUS$]	Gap [%]	Time [s]
I8	4×4	Yes	-	Yes	64.44	1771	1716	3.21	2252
I9	4×4	Yes	3–4	Yes	60.88	1809	1759	2.84	2384
I10	4×4	Yes	7–8	Yes	64.24	1817	1762	3.12	2244
I11	4×4	Yes	11–12	Yes	64.42	1824	1769	3.11	2333

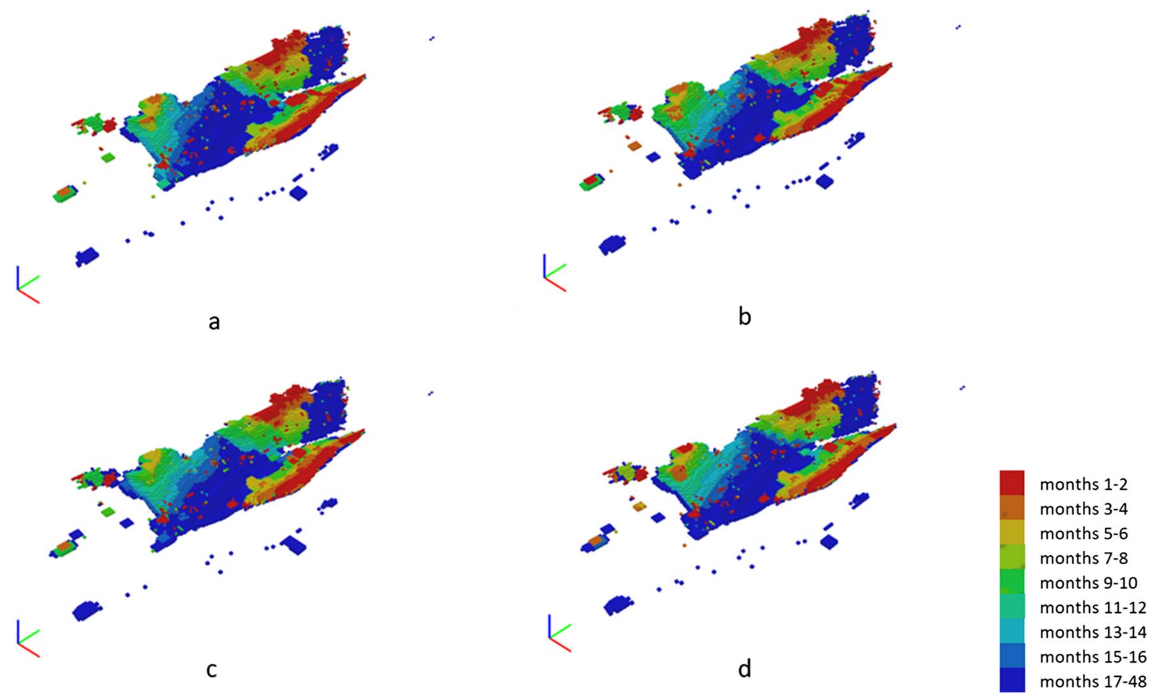


Fig. 8 Illustration of the spatial distribution of the first 18 months of production. **a** Production adjusted + UG case, **b** M1UG case, **c** M2UG case, and **d** M3UG case

4 Conclusions

This paper presents a novel approach to addressing the mining width-constrained open-pit production scheduling mine planning problem. A mathematical formulation is introduced to effectively integrate mining width constraints into the production scheduling process. To address the complexity of large-scale instances, we propose a solution method that solves a partly relaxed version of the formulation using the BZ algorithm and adapting the mining width template variable fixing proposed in Yarmuch et al. (2021b). The proposed algorithm is tested on real-world Radomiro Tomic medium-term mine planning scenarios, encompassing diverse constraints such as multiple ore material destinations, maximum sinking rate, and production constraints.

By solving multiple instances based on RT dataset, the study evaluates the performance of the developed algorithms in solving problems of varying complexities. The obtained solutions of the MW-OPMPS exhibit a notable improvement in mining width satisfiability, increasing from 2 to 60% as compared to the OPMPS. The introduced model's efficacy is further demonstrated by an optimality gap consistently maintained within 4% for all instances.

Furthermore, the research offers practical insights into real-world decision-making processes. Even though solutions still need to be reworked to make them practical, enable engineers to conduct trade-off studies, such as optimizing the timing of major truck maintenance. Moreover, the impact of external factors, like the delay in the production of the Chuquicamata underground project, is quantitatively assessed, enabling informed decisions in Radomiro Tomic's planning.

Future research can further enhance the solution method and optimization models by incorporating additional operational and economic parameters to address the complexities of mining equipment operations and the inclusion of dynamic stockpiles. Note that the values presented have been scaled due to confidentiality considerations.

Appendix. Formulation of $\mathcal{P}^*$

The following describes the formulation of the subproblem $\mathcal{P}^*$; we use the same notation and sets described in Sect. 2.

Variables.

The set of variables are associated with the extraction of the blocks.

$$z_{b,d,t} = \begin{cases} 1, & \text{if block } b \text{ is sent to destination d by period t} \\ 0, & otherwise \end{cases}$$

Objective Function

$$\max \sum_{b\in\mathcal{B}} \sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} p^*_{b,d,t} \cdot (z_{b,d,t} - z_{b,d,t-1})$$

The objective function aims to maximize the discounted cashflow or net present value.

Constraints

$$\sum_{b\in\mathcal{B}} a_{b,d} \cdot (z_{b,d,t} - z_{b,d,t-1}) \leq C_{d,t} \;\; t \in \mathcal{T}, d \in \mathcal{D} \tag{8}$$

$$z_{a,D,t} \leq z_{b,D,t} \;\; (a,b) \in \mathcal{A} \cup \mathcal{H}, t \in \mathcal{T} \tag{9}$$

$$z_{b,D,t} \leq z_{b,0,t+1} \;\; b \in \mathcal{B}, t \in \{1,..,T-1\} \tag{10}$$

$$z_{b,d,t} \leq z_{b,d+1,t} \;\; b \in \mathcal{B}, t \in \mathcal{T}, d \in \{1,..,D-1\} \tag{11}$$

$$z_{b,d,0} = 0 \;\; b \in \mathcal{B}, d \in \mathcal{D} \tag{12}$$

$$z_{i,D,t} \leq z_{j,0,t+1} \;\; (i,j) \in \mathcal{S} \tag{13}$$

$$\sum_{d\in\mathcal{D}} \sum_{t\in\mathcal{T}} (z_{b,d,t} - z_{b,d,t-1}) \leq 1 \;\; b \in \mathcal{B} \tag{14}$$

Constraint (8) represents the resource constraint per destination per period; in our case, the resource associated with each block consists of its tonnage. This constraint can directly be extended to multiple resource constraints per destination. Constraint (9) ensures that, to extract a block b by period t, all its vertical and horizontal predecessors must also be extracted by period t. Constraint (10) forces that if a block is mined by a period t, then it will continue as mined for all subsequent periods. Constraint (11) is analogue to Constraint (10) but regarding the destinations. Constraint (12) ensures that no blocks can be extracted at period 0. Constraint (13) ensures that all the blocks that could violate the sinking rate constraint from block i are extracted at least a period earlier than block j itself. Constraint (14) ensures that the blocks are selected on at most one period one destination.

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