$$V(x) = \frac{A}{x^2} - \frac{B}{x} \quad , \quad A, B > 0$$

a) $[V] = J = kg\,\frac{m^2}{s^2} \Rightarrow \left[\frac{A}{x^2}\right] = \frac{[A]}{m^2} = kg\,\frac{m^2}{s^2} \Rightarrow [A] = \frac{kg\,m^4}{s^2}$ +0,5

+0,5

1,5)

y $\frac{[B]}{m} = kg\,\frac{m^2}{s^2} \Rightarrow [B] = \frac{kg\,m^3}{s^2}$ +0,5

b) $F = -\nabla V = -\frac{\partial V}{\partial x} = -\left(-\frac{2A}{x^3} + \frac{B}{x^2}\right) = \frac{2A}{x^3} - \frac{B}{x^2}$ +0,5

+1

2)

$F = ma \Rightarrow a(x) = \frac{F}{m} = \frac{1}{m}\left(\frac{2A}{x^3} - \frac{B}{x^2}\right)$ +0,5

c) Ptos de eq $F = 0 \Rightarrow F = \frac{2A - Bx}{x^3} = 0 \Rightarrow x^* = \frac{2A}{B}$ +0,5

1,5)

Frec. peq osc. $\omega^2 = \frac{V''(x^*)}{m}$; $V'' = \frac{6A}{x^4} - \frac{2B}{x^3}$ +0,5

$$\Rightarrow \omega^2 = \frac{V''(x^*)}{m} = \frac{1}{m}\left(\frac{6A}{x^{*4}} - \frac{2B}{x^{*3}}\right) = \frac{2}{m}\,\frac{3A - Bx^*}{x^{*4}}$$ +0,5

$$= \frac{2}{m}\left(\frac{B}{2A}\right)^4\left(3A - B\left(\frac{2A}{B}\right)\right) = \frac{2}{m}\left(\frac{B}{2A}\right)^4 A$$

$B^4/8mA^3$

$\sim \frac{1}{kg} \cdot \frac{1}{m^4} \cdot \frac{kg\,m^4}{s^2} \sim \frac{1}{s^2}$ ←

d) $E = \frac{1}{2}m\dot{x}^2 + V(x)$. Por conservación de energía:

1)

$E_i = V(x^*/2) = E_f = \frac{1}{2}m\dot{x}_f^2 + V(x^*)$ +0,5

$$\Rightarrow \dot{x}_f^2 = \frac{2}{m}\left(V(x^*/2) - V(x^*)\right) = \frac{2}{m}\left[\left(\frac{A}{\left(\frac{A}{B}\right)^2} - \frac{B}{\left(\frac{A}{B}\right)}\right) - \left(\frac{A}{\left(\frac{2A}{B}\right)^2} - \frac{B}{\left(\frac{2A}{B}\right)}\right)\right]$$

$$= \frac{2}{m}\left[\frac{B^2}{A} - \frac{B^2}{A} - \frac{B^2}{4A} + \frac{B^2}{2A}\right] = \frac{1}{m}\,\frac{B^2}{2A}$$ +0,5

$\frac{1}{kg}\,\frac{kg^2 m^6/s^4}{kg\,m^4/s^2} \sim \frac{m^2}{s^2}$ ←