

10

pt. Sea $T: \mathbb{P}_2 \rightarrow \mathbb{P}_2$ def. por $T(a_2x^2 + a_1x + a_0) = (a_0 + 2a_2)x^2 + a_1x + (2a_0 + a_2)x^3$

a) Demuestre que es lineal

Sia $p, q \in \mathbb{P}_2$ y $\lambda \in \mathbb{R}$ polo: $T(p + \lambda q) = T(p) + \lambda T(q)$

$$\hookrightarrow p = a_2x^2 + a_1x + a_0$$

donde:

$$p + \lambda q = \underbrace{(a_2 + \lambda b_2)}_{c_2}x^2 + \underbrace{(a_1 + \lambda b_1)}_{c_1}x + \underbrace{(a_0 + \lambda b_0)}_{c_0}$$

$$\begin{aligned} T(p + \lambda q) &= (c_0 + 2c_2) + c_1x + (2c_0 + c_2)x^3 \\ &= [(a_0 + \lambda b_0) + 2(a_2 + \lambda b_2)] + (a_1 + \lambda b_1)x + [2(a_0 + \lambda b_0) + (a_2 + \lambda b_2)]x^2 \\ &= (a_0 + 2a_2) + (\lambda b_0 + 2\lambda b_2) + a_1x + \lambda b_1x + (2a_0 + a_2)x^2 + (\lambda b_0 + 2\lambda b_2) + \lambda b_1x + (2\lambda b_0 + \lambda b_2)x^2 \\ &= T(p) + \lambda [(b_0 + 2b_2) + b_1x + (2b_0 + b_2)x^2] \\ &= T(p) + \lambda T(q) \end{aligned}$$

b) Encuentre la matriz representante. Base canónica: $\{1, x, x^2\}$
de \mathbb{P}_2

$$1 = \underbrace{0 \cdot x^2}_{a_2} + \underbrace{0 \cdot x}_{a_1} + \underbrace{1 \cdot 1}_{a_0} \mapsto \text{Como polinomio}$$

$$\begin{aligned} \therefore T(1) &= (1 + 2 \cdot 0) + 0 \cdot x + (2 \cdot 1 + 0)x^2 \\ &= 1 + 2 \cdot x^2 = \underbrace{1 \cdot 1}_{x} + \underbrace{0 \cdot x}_{x} + \underbrace{2 \cdot x^2}_{x^2} \end{aligned}$$

$$x = \underbrace{0 \cdot x^2}_{a_2} + \underbrace{1 \cdot x}_{a_1} + \underbrace{0 \cdot 1}_{a_0} \mapsto \text{Como polinomio}$$

$$\begin{aligned} \therefore T(x) &= (0 + 2 \cdot 0) + 1 \cdot x + (2 \cdot 0 + 0)x^2 \\ &= x = \underbrace{0 \cdot 1}_{x} + \underbrace{1 \cdot x}_{x} + \underbrace{0 \cdot x^2}_{x^2} \end{aligned}$$

$$x^2 = \underbrace{1 \cdot x^2}_{a_2} + \underbrace{0 \cdot x}_{a_1} + \underbrace{0 \cdot 1}_{a_0} \mapsto \text{Como polinomio}$$

$$\begin{aligned} \therefore T(x^2) &= (0 + 2 \cdot 1) + 0 \cdot x + (2 \cdot 0 + 1)x^2 \\ &= 2 + x^2 = \underbrace{2 \cdot 1}_{x^2} + \underbrace{0 \cdot x}_{x} + \underbrace{1 \cdot x^2}_{x^2} \end{aligned}$$

\therefore la matriz Rep.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

2. Sea $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ definido por: $T \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x+y+z+w \\ 2x+y-z \\ x-y+z-w \end{pmatrix}$

a) Matriz representante en bases canónicas.

$$T \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0+0+0 \\ 2 \cdot 1 + 0 - 0 \\ 1-0+0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \underline{1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{2} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+1+0+0 \\ 2 \cdot 0 + 1 - 0 \\ 0-1+0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = \underline{1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{-1} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+0+1+0 \\ 2 \cdot 0 + 0-1 \\ 0-0+1-0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \underline{-1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{-1} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+0+0+1 \\ 2 \cdot 0 + 0-0 \\ 0-0+0-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \underline{1} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{0} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{-1} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

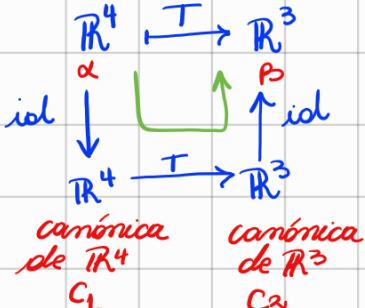
o) la matriz Represent.

$$[T]_{\xi_2 \alpha}^{\beta} = \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & -1 & 1 & -1 \end{array} \right]$$

3×4

pues las demás
son más
sencillas

b)



$$[T]_{\beta \alpha}^{\gamma} = [id]_{\beta \alpha}^{\gamma} [T]_{\xi_2 \alpha}^{\beta} [id]_{\xi_2 \alpha}^{\gamma}$$

c) ¿Qué es la función $id: A \rightarrow A$?

Hacemos, $id(a) = a$

$$e_i = \begin{pmatrix} 0 \\ \vdots \\ i \\ \vdots \\ 0 \end{pmatrix} \xrightarrow{\text{posic } i}$$

o) ya tenemos $[T]_{\xi_2 \alpha}^{\beta}$, falta las identidades

$$1^{\circ}) [id]_{\alpha \alpha}^{\alpha}; id \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underline{1} \cdot e_1 + \underline{0} \cdot e_2 + \underline{0} \cdot e_3 + \underline{0} \cdot e_4$$

$$\text{id} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underline{1} \cdot e_1 + \underline{1} \cdot e_2 + \underline{0} \cdot e_3 + \underline{0} \cdot e_4$$

$$\text{id} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \underline{1} \cdot e_1 + \underline{1} \cdot e_2 + \underline{1} \cdot e_3 + \underline{0} \cdot e_4$$

$$\text{id} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \underline{1} \cdot e_1 + \underline{1} \cdot e_2 + \underline{1} \cdot e_3 + \underline{1} \cdot e_4$$

$$\therefore [\text{id}]_{\beta \leftarrow \alpha} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$2) [\text{id}]_{\beta \leftarrow \alpha} ; \quad \text{id} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \underline{1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{0} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \underline{0} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{id} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \underline{-1} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{1} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \underline{0} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{id} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \underline{0} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{-1} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \underline{1} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\therefore [\text{id}]_{\beta \leftarrow \alpha} ; \quad \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

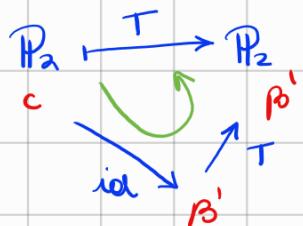
$$\therefore [T]_{\beta \leftarrow \alpha} = [\text{id}]_{\beta \leftarrow \alpha} [+]_{\xi \leftarrow \alpha} [\text{id}]_{\xi \leftarrow \alpha}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$3 \times 3 \quad 3 \times 4 \quad 4 \times 4 = 3 \times 4$$

p3. $T: P_2 \mapsto P_2$ Calculate $[T]_{\beta \leftarrow \beta}$

podemos
navegar
trigonometria



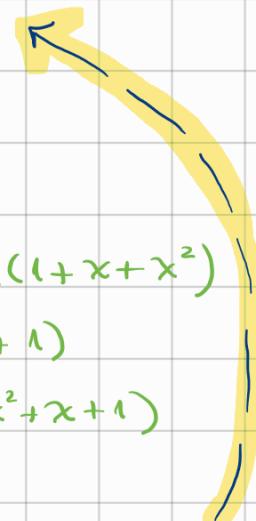
$$\begin{aligned} \text{id}(1) &= 1 = \underline{1} \cdot 1 + \underline{0} \cdot (x+1) + \underline{0} \cdot (x^2+x+1) \\ \text{id}(x) &= x = \underline{-1} \cdot 1 + \underline{1} \cdot (x+1) + \underline{0} \cdot (x^2+x+1) [\text{id}]_{\beta' \beta} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ \text{id}(x^2) &= x^2 = \underline{0} \cdot 1 + \underline{-1} \cdot (x+1) + \underline{1} \cdot (x^2+x+1) \end{aligned}$$

$$T(1) = 3 + 2x + 4x^2 = \underline{1} \cdot 1 + \underline{-2} \cdot (1+x) + \underline{4} \cdot (1+x+x^2)$$

$$\begin{aligned} T(x+1) &= T(x) + T(1) = (3 + 2x + 4x^2) + (2 + 2x^2) \\ &= (5 + 2x + 6x^2) = \underline{3} \cdot 1 + \underline{-4} \cdot (1+x) + \underline{6} \cdot (1+x+x^2) \end{aligned}$$

$$\begin{aligned} T(x^2+x+1) &= T(x^2) + T(x) + T(1) \\ &= (4 + 2x + 3x^2) + [5 + 2x + 6x^2] \\ &= (9 + 4x + 9x^2) = \underline{5} \cdot 1 + \underline{-5} \cdot (1+x) + \underline{9} \cdot (1+x+x^2) \end{aligned}$$

$$\therefore [T]_{\beta' \beta} = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -4 & -5 \\ 4 & 6 & 9 \end{bmatrix}$$

$$\begin{aligned} \therefore [T]_{\beta' \beta}^{\leftarrow} &= [T]_{\beta' \beta}^{\leftarrow} \cdot [\text{id}]_{\beta' \beta}^{\leftarrow} = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -4 & -5 \\ 4 & 6 & 9 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 2 \\ -2 & -2 & -1 \\ 4 & 2 & 3 \end{bmatrix} \end{aligned}$$


→ otra forma de hacerlo es hacerla de esta:

$$T(1) = 3 + 2x + 4x^2 = \underline{1} \cdot 1 + \underline{-2} \cdot (1+x) + \underline{4} \cdot (1+x+x^2)$$

$$T(x) = 2 + 2x^2 = \underline{2} \cdot 1 + \underline{-2} \cdot (x+1) + \underline{2} \cdot (x^2+x+1)$$

$$T(x^2) = 4 + 2x + 3x^2 = \underline{2} \cdot 1 + \underline{-1} \cdot (x+1) + \underline{3} \cdot (x^2+x+1)$$

∴

$$[T]_{\beta' \beta}^{\leftarrow} = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -2 & -1 \\ 4 & 2 & 3 \end{bmatrix} \quad \sim \text{es lo mismo}$$