p4. Sea $T: \mathbb{P}_2 \to \mathbb{P}_2$ def. por $T(a_2x^2 + a_1x + a_0) = (a_0 + 2a_2) + a_1x + (2a_0 + a_2)x^2$

a) Demuestre que es lineal

Sea $p, q \in \mathbb{P}_2$ y $\lambda \in \mathbb{R}$ pdq: $T(p + \lambda q) = T(p) + \lambda T(q)$

$\hookrightarrow$ $p = a_2x^2 + a_1x + a_0$ $\quad$ $q = b_2x^2 + b_1x + b_0$

donde:

$$p + \lambda q = \overbrace{(a_2 + \lambda b_2)}^{c_2}x^2 + \overbrace{(a_1 + \lambda b_1)}^{c_1}x + \overbrace{(a_0 + \lambda b_0)}^{c_0}$$

$$\begin{aligned}
T(p + \lambda q) &= (c_0 + 2c_2) + c_1x + (2c_0 + c_2)x^2 \\
&= [(a_0 + \lambda b_0) + 2(a_2 + \lambda b_2)] + (a_1 + \lambda b_1)x + [2(a_0 + \lambda b_0) + (a_2 + \lambda b_2)]x^2 \\
&= (a_0 + 2a_2) + (\lambda b_0 + 2\lambda b_2) + a_1x + \lambda b_1x + (2a_0 + a_2)x^2 + (2\lambda b_0 + \lambda b_2)x^2 \\
&= (a_0 + 2a_2) + a_1x + (2a_0 + a_2)x^2 + (\lambda b_0 + 2\lambda b_2) + \lambda b_1x + (2\lambda b_0 + \lambda b_2)x^2 \\
&= T(p) + \lambda[(b_0 + 2b_2) + b_1x + (2b_0 + b_2)x^2] \\
&= T(p) + \lambda T(q)
\end{aligned}$$

b) Encuentre la matriz representante. Base canónica: $\{1, x, x^2\}$ de $\mathbb{P}_2$

$1 = \underbrace{0}_{a_2} \cdot x^2 + \underbrace{0}_{a_1} \cdot x + \underbrace{1}_{a_0} \cdot 1 \mapsto$ Como polinomio

$\therefore$ $\boxed{T(1) =}$ $(1 + 2 \cdot 0) + 0 \cdot x + (2 \cdot 1 + 0)x^2$

$= 1 + 2 \cdot x^2 = \underline{1} \cdot 1 + \underline{0} \cdot x + \underline{2} \cdot x^2$

$x = \underbrace{0}_{a_2} \cdot x^2 + \underbrace{1}_{a_1} \cdot x + \underbrace{0}_{a_0} \cdot 1 \mapsto$ Como polinomio

$\therefore$ $\boxed{T(x) =}$ $(0 + 2 \cdot 0) + 1 \cdot x + (2 \cdot 0 + 0)x^2$

$= x = \underline{0} \cdot 1 + \underline{1} \cdot x + \underline{0} \cdot x^2$

$x^2 = \underbrace{1}_{a_2} \cdot x^2 + \underbrace{0}_{a_1} \cdot x + \underbrace{0}_{a_0} \cdot 1 \mapsto$ Como polinomio

$\therefore$ $\boxed{T(x^2) =}$ $(0 + 2 \cdot 1) + 0 \cdot x + (2 \cdot 0 + 1)x^2$

$= 2 + x^2 = \underline{2} \cdot 1 + \underline{0} \cdot x + \underline{1} \cdot x^2$

$\therefore$ la matriz Rep.

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

p2. Sea $T: \mathbb{R}^4 \to \mathbb{R}^3$ definido por: $T\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x+y+z+w \\ 2x+y-z \\ x-y+z-w \end{pmatrix}$

a) matriz representante en bases canonicas.

$$T\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1+0+0+0 \\ 2\cdot 1+0-0 \\ 1-0+0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \underline{1}\cdot\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{2}\cdot\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{1}\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+1+0+0 \\ 2\cdot 0+1-0 \\ 0-1+0-0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \underline{1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0+0+1+0 \\ 2\cdot 0+0-1 \\ 0-0+1-0 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \underline{1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{-1}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$T\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0+0+0+1 \\ 2\cdot 0+0-0 \\ 0-0+0-1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \underline{1}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \underline{0}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \underline{-1}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore$ la matriz Represent.

$$[T]_{C_2 \leftarrow C_1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix}^{3\times 4}$$

b)

$$\begin{array}{ccc} \mathbb{R}^4_{\alpha} & \xrightarrow{T} & \mathbb{R}^3_{\beta} \\ id \downarrow & & \uparrow id \\ \mathbb{R}^4 & \xrightarrow{T} & \mathbb{R}^3 \end{array}$$

canónica de $\mathbb{R}^4$ C_1 canónica de $\mathbb{R}^3$ C_2

pues es más sencillo

$$[T]_{\beta \leftarrow \alpha} = [id]_{\beta \leftarrow C_2}\,[T]_{C_2 \leftarrow C_1}\,[id]_{C_1 \leftarrow \alpha}$$

¿Que es la función $id: A \to A$?

$\forall a \in A$, $id(a) = a$

$\therefore$ ya tenemos $[T]_{C_2 \leftarrow C_1}$ falta las identidades $e_i = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \end{pmatrix}$ → posic i

1°) $[id]_{C_1 \leftarrow \alpha}$; $id\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underline{1}\cdot e_1 + \underline{0}\cdot e_2 + \underline{0}\cdot e_3 + \underline{0}\cdot e_4$

$$id\begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} = \underline{1}\cdot e_1 + \underline{1}\cdot e_2 + \underline{0}\cdot e_3 + \underline{0}\cdot e_4$$

$$id\begin{pmatrix}1\\1\\1\\0\end{pmatrix} = \begin{pmatrix}1\\1\\1\\0\end{pmatrix} = \underline{1}\cdot e_1 + \underline{1}\cdot e_2 + \underline{1}\cdot e_3 + \underline{0}\cdot e_4$$

$$id\begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\1\\1\end{pmatrix} = \underline{1}\cdot e_1 + \underline{1}\cdot e_2 + \underline{1}\cdot e_3 + \underline{1}\cdot e_4$$

$$\therefore \quad [id]_{C_1 \leftarrow \alpha} = \begin{bmatrix}1&1&1&1\\0&1&1&1\\0&0&1&1\\0&0&0&0\end{bmatrix}$$

2°) $[id]_{\beta \leftarrow C_2}$;

$$id\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix} = \underline{1}\begin{pmatrix}1\\0\\0\end{pmatrix} + \underline{0}\begin{pmatrix}1\\1\\0\end{pmatrix} + \underline{0}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$id\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\0\end{pmatrix} = \underline{-1}\begin{pmatrix}1\\0\\0\end{pmatrix} + \underline{1}\begin{pmatrix}1\\1\\0\end{pmatrix} + \underline{0}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$id\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} = \underline{0}\begin{pmatrix}1\\0\\0\end{pmatrix} + \underline{-1}\begin{pmatrix}1\\1\\0\end{pmatrix} + \underline{1}\begin{pmatrix}1\\1\\1\end{pmatrix}$$

$$\therefore \quad [id]_{\beta \leftarrow C_2} \; ; \; \begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$$\therefore \quad [T]_{\beta \leftarrow \alpha} = [id]_{\beta \leftarrow C_2}\,[T]_{C_2 \leftarrow C_1}\,[id]_{C_1 \leftarrow \alpha}$$

$$= \begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&1&1\\2&1&-1&0\\1&-1&1&-1\end{bmatrix}\begin{bmatrix}1&1&1&1\\0&1&1&1\\0&0&1&1\\0&0&0&1\end{bmatrix}$$

$$3\times3 \quad 3\times4 \quad 4\times4 \quad = 3\times4$$

p3. $T: \mathbb{P}_2 \longmapsto \mathbb{P}_2$ Calcule $[T]_{\beta' \leftarrow \beta}$

podemos hacer triangulos

$$\begin{tikzcd}
\mathbb{P}_2 \; C \arrow[rr, "T"] \arrow[dr, "id"'] & & \mathbb{P}_2 \; \beta' \\
& \beta' \arrow[ur, "T"'] &
\end{tikzcd}$$

$$id(1) = 1 = 1 \cdot 1 + 0\,(x+1) + 0\,(x^2+x+1)$$
$$id(x) = x = -1 \cdot 1 + 1\,(x+1) + 0\,(x^2+x+1)$$
$$id(x^2) = x^2 = 0 \cdot 1 + -1\,(x+1) + 1\,(x^2+x+1)$$

$$[id]_{\beta'\beta} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(1) = 3+2x+4x^2 = 1 \cdot 1 + -2\,(1+x) + 4 \cdot (1+x+x^2)$$
$$T(x+1) = T(x) + T(1) = (3+2x+4x^2) + (2+2x^2)$$
$$= (5+2x+6x^2) = 3 \cdot 1 + -4\,(1+x) + 6 \cdot (1+x+x^2)$$
$$T(x^2+x+1) = T(x^2) + T(x) + T(1)$$
$$= (4+2x+3x^2) + [5+2x+6x^2]$$
$$= (9+4x+9x^2) = 5 \cdot 1 + -5\,(1+x) + 9 \cdot (1+x+x^2)$$

$$\therefore\ [T]_{\beta'\beta'} = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -4 & -5 \\ 4 & 6 & 9 \end{bmatrix}$$

$$\therefore\ [T]_{\overleftarrow{\beta'\beta}} = [T]_{\overleftarrow{\beta'\beta'}} \cdot [id]_{\overleftarrow{\beta'\beta}} = \begin{bmatrix} 1 & 3 & 5 \\ -2 & -4 & -5 \\ 4 & 6 & 9 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 2 & 2 \\ -2 & -2 & -1 \\ 4 & 2 & 3 \end{bmatrix}$$

↦ otra forma de hacerlo es hacerla de una:

$$T(1) = 3+2x+4x^2 = 1 \cdot 1 + -2\,(1+x) + 4\,(1+x+x^2)$$
$$T(x) = 2+2x^2 = 2 \cdot 1 + -2\,(x+1) + 2\,(x^2+x+1)$$
$$T(x^2) = 4+2x+3x^2 = 2 \cdot 1 + -1\,(x+1) + 3\,(x^2+x+1)$$

$$\therefore\ [T]_{\overleftarrow{\beta'\beta}} = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -2 & -1 \\ 4 & 2 & 3 \end{bmatrix}$$ ⇝ es lo mismo