p1) $A = \begin{pmatrix} -1 & 2 \\ 2 & 1 \end{pmatrix}$ $\lambda_1 = \sqrt{5}$ $\lambda_2 = -\sqrt{5}$

$\lambda_1 = \sqrt{5}$ $\begin{pmatrix} -1-\sqrt{5} & 2 \\ 2 & 1-\sqrt{5} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

pues quiero encontrar $v = \begin{pmatrix} x \\ y \end{pmatrix}$ tq:

$$A \cdot v = \lambda \cdot v$$
$$\Rightarrow (A - I\lambda) v = 0$$

1) $x(-1-\sqrt{5}) + 2y = 0$

2) $2x + (1-\sqrt{5}) y = 0$

de 1) $-x(1+\sqrt{5}) + 2y = 0$

$$\Rightarrow y = \frac{x(1+\sqrt{5})}{2}$$

en 2) $2x + (1-\sqrt{5}) \frac{(1+\sqrt{5})}{2} x = 0$

$$2x + \frac{(1-5)}{2} x = 0$$

$$2x + \frac{-4}{2} x = 0 \checkmark$$

no hay más info $\therefore$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x(1+\sqrt{5})/2 \end{pmatrix} = x \begin{pmatrix} 1 \\ (1+\sqrt{5})/2 \end{pmatrix}$$

$\lambda_2 = \sqrt{5}$ $\begin{pmatrix} -1+\sqrt{5} & 2 \\ 2 & 1+\sqrt{5} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

1) $x(\sqrt{5}-1) + 2y = 0$ $\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-(\sqrt{5}+1)}{2} y \\ y \end{pmatrix}$

2) $2x + (\sqrt{5}+1) y = 0$

de 2) $x = -\frac{(\sqrt{5}+1)}{2} y$

$$\therefore \quad v = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -(\sqrt{5}+1)/2 \\ 1 \end{pmatrix} y$$

$$\therefore \quad W_{\lambda_1} = \left\langle \begin{pmatrix} 1 \\ (1+\sqrt{5})/2 \end{pmatrix} \right\rangle \qquad W_{\lambda_2} = \left\langle \begin{pmatrix} -\frac{(\sqrt{5}+1)}{2} \\ 1 \end{pmatrix} \right\rangle$$

$$B = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix} \qquad \begin{array}{l} \lambda_1 = 1 \\ \lambda_2 = 2 \end{array} \qquad \lambda_3 = 3$$

$\lambda_1 = 1$

$$\begin{pmatrix} 0 & 2 & -1 \\ 1 & -1 & 1 \\ 4 & -4 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1) $2y - z = 0$
2) $x - y + z = 0$
3) $4x - 4y + 4z = 0$

de 1) $z = 2y$

en 2) $x - y + 2y = 0$

$x + y = 0$

$\Rightarrow x = -y$

en 3) no entrega info

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ 2y \end{pmatrix} = y \underbrace{\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}}_{\vec{v_p}} \qquad W_{\lambda_1} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \right\rangle$$

$\lambda_2 = 2$

$$\begin{pmatrix} -1 & 2 & -1 \\ 1 & -2 & 1 \\ 4 & -4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1) $-x + 2y - z = 0$
2) $x - 2y + z = 0$
3) $4x - 4y + 3z = 0$

1) = − 2)

de 2) $x = 2y - z$

en 3) $4(2y - z) - 4y + 3z = 0$

$8y - 4z - 4y + 3z = 0$

$4y - z = 0$

$\Rightarrow \boxed{z = 4y}$

$y \quad x = 2y - 4y = -2y$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2y \\ y \\ 4y \end{pmatrix} = y \underbrace{\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}}_{v_p}$$

$$y \; W_{\lambda_2} = \left\langle \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \right\rangle$$

$\boxed{\lambda_3 = 3}$

$$\begin{pmatrix} -2 & 2 & -1 \\ 1 & -3 & 1 \\ 4 & -4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left.\begin{array}{l} 1) \; -2x + 2y - z = 0 \\ 2) \; 1x - 3y + z = 0 \\ 3) \; 4x - 4y + 2z = 0 \end{array}\right\} (-2 \cdot 1) = 3)$$

de 1) $z = 2y - 2x$

en 2) $x - 3y + 2y - 2x = 0$

$-x - y = 0$

$\Rightarrow \boxed{x = -y} \Rightarrow \boxed{z = 4y}$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ 4y \end{pmatrix} = y \underbrace{\begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix}}_{v_p} \qquad W_{\lambda_3} = \left\langle \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} \right\rangle$$

$$C = \begin{pmatrix} 2 & 1 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \lambda_1 = 1 \qquad \lambda_2 = 2$$

$\boxed{\lambda_1 = 1}$

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

1) $x + y + w = 0$ 3) $z + w = 0$

2) $y + z = 0$ $\Rightarrow \boxed{w = -z}$

en 2) $\boxed{y = -z}$ $\therefore$ $x - 2z = 0$

$\boxed{x = 2z}$

NW

$$\therefore \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 2z \\ -z \\ z \\ -z \end{pmatrix} = z \underbrace{\begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix}}_{\vec{v_p}} \quad W_{\lambda_1} = \left\langle \begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix} \right\rangle$$

$\boxed{\lambda_2 = 2}$

$$\begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

1) $y + w = 0$ 2) $\boxed{z = 0}$ 3) $w = 0$
4) $\boxed{w = 0}$

$\Rightarrow \boxed{y = 0}$

¿Qué pasa con x? ¡es libre!

$$\therefore \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \\ 0 \end{pmatrix} = x \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\vec{v_p}} \therefore W_{\lambda_2} = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\rangle$$

p2) $\begin{pmatrix} 1 & 2 & \alpha \\ 2 & 1 & \beta \\ 2 & 2 & \gamma \end{pmatrix}$

a) Sabemos que $\lambda = 3$ y su vector propio es $(1,1,1)^T$

Sabemos que $A \cdot v = \lambda v$

$$\therefore \begin{pmatrix} 1 & 2 & \alpha \\ 2 & 1 & \beta \\ 2 & 2 & \gamma \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

1) $1 + 2 + \alpha = 3$

2) $2 + 1 + \beta = 3$

3) $2 + 2 + \gamma = 3$

$\therefore \quad \alpha = 0 \quad \beta = 0 \quad \gamma = -1$

$$\therefore \quad A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}$$

b) Calcular vp y $\vec{vp}$ restantes.

$$\det\left(\begin{pmatrix} 1-\lambda & 2 & 0 \\ 2 & 1-\lambda & 0 \\ 2 & 2 & -1-\lambda \end{pmatrix}\right) = p(\lambda) = 0$$

puedo elegir filas como columnas! escojamos la 3° columna pues tiene 2 ceros!

$(x+a)(x+b) = (x^2 + (a+b)x + a \cdot b)$

$$+(-1-\lambda) \det\begin{pmatrix} 1-\lambda & 2 \\ 2 & 1-\lambda \end{pmatrix} = 0$$

$$-(1+\lambda) \cdot \left[(1-\lambda)(1-\lambda) - 4 \right] = 0$$

$$-(1+\lambda) \left[(\lambda^2 - 2\lambda + 1) - 4 \right] = 0$$

$$-(1+\lambda) \left[\lambda^2 - 2\lambda - 3 \right] = 0$$

$$-(1+\lambda)(\lambda-3)(\lambda+1) = 0$$

i.e. $\lambda_1 = -1$ y $\lambda_2 = 3$ $\square$

$\underline{\lambda_1 = -1}$

$$\therefore \begin{pmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

1) $2x + 2y = 0$

$\Rightarrow \boxed{x = -y}$

de z no sabemos! es libre

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y \\ y \\ z \end{pmatrix} = y \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

124

$\therefore$ $\vec{v_p}$ son $\begin{pmatrix}-1\\1\\0\end{pmatrix}$ y $\begin{pmatrix}0\\0\\1\end{pmatrix}$ para $\lambda=-1$

son li $\therefore$ $W_{\lambda=2} = \left\langle \begin{pmatrix}-1\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix} \right\rangle$

$= \ker(A+\lambda I)$

p3) $\begin{pmatrix}1 & -1\\ x_1 & x_2\end{pmatrix}$ sus $\vec{vp}$ asociados son $\begin{pmatrix}1\\1\end{pmatrix}$ y $\begin{pmatrix}1/2\\1\end{pmatrix}$

$\exists\, \lambda_1$ tq;

$$\begin{pmatrix}1 & -1\\ x_1 & x_2\end{pmatrix} \cdot \begin{pmatrix}1\\1\end{pmatrix} = \lambda_1 \begin{pmatrix}1\\1\end{pmatrix}$$

$1 + -1 = \lambda_1 \cdot 1 \to \boxed{0 = \lambda_1}$

$x_1 + x_2 = \lambda_1 \cdot 1$

$\therefore$ $\boxed{x_1 = -x_2}$

$\exists\, \lambda_2$ tq; $\begin{pmatrix}1 & -1\\ x_1 & x_2\end{pmatrix}\begin{pmatrix}1/2\\1\end{pmatrix} = \lambda_2 \begin{pmatrix}1/2\\1\end{pmatrix}$

1) $1/2 - 1 = \lambda_2 \cdot 1/2$

2) $x_1/2 + x_2 = \lambda_2$

de 1) tengo que;

$-1/2 = \lambda_2/2 \Rightarrow \boxed{\lambda_2 = -1}$

luego de 2) $x_1 + 2x_2 = \underbrace{2\lambda_2}_{=-2}$

pero sabemos que $x_1 = -x_2$

$\therefore \quad -x_2 + 2x_2 = -2$

$\boxed{x_2 = -2} \quad \therefore \quad \boxed{x_1 = 2}$

$\therefore$ la matriz es: $\begin{pmatrix} 1 & -1 \\ 2 & -2 \end{pmatrix}$