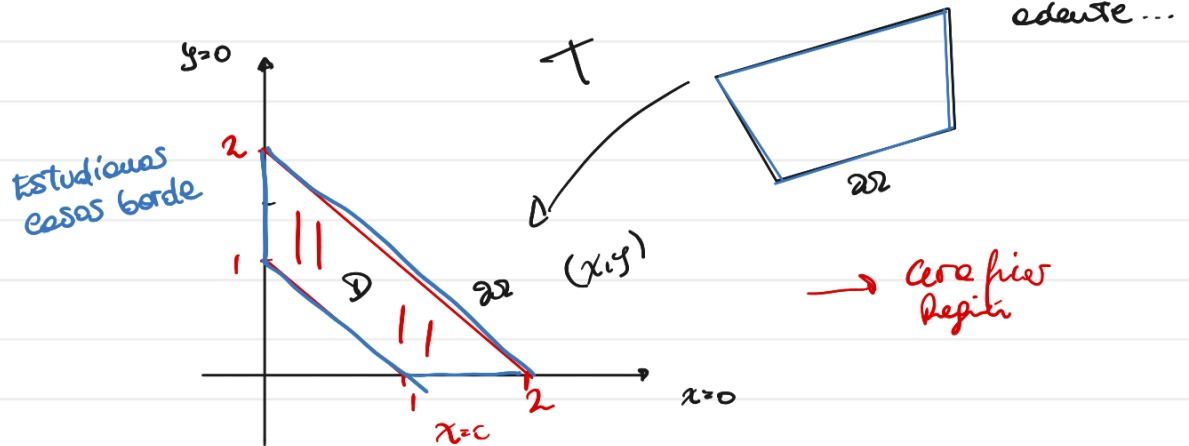


P1.- (Total puntaje P1=1 punto) Calcular la integral

$$\iint_D \cos\left(\frac{y-x}{y+x}\right),$$

donde D es la región delimitada por las rectas $y=0$, $x=0$, $x+y=1$ y $x+y=2$. \rightarrow Casos Borde

Grificamos la región D



Consideremos el CV $u = y - x$, $v = y + x$,

Notamos que, $x = \frac{1}{2}(v-u)$, $y = \frac{1}{2}(u+v)$

\rightarrow Escribir transf.

$$T(u,v) = \left(\underbrace{\frac{1}{2}(v-u)}_{x(u,v)}, \underbrace{\frac{1}{2}(u+v)}_{y(u,v)} \right)$$

\rightarrow Encontrar $|\det DT(u,v)|$

luego

$$DT(u,v) = \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{pmatrix} = \begin{pmatrix} -1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

$$\Rightarrow |\det DT(u,v)| = 1/2$$

Buscamos escribir la región en términos de (u,v)

TCU

$$\int_{T(D)} f(y) dy = \int_{D'} (f \circ T)(x) |\det DT(x)| dx$$

$D \leftrightarrow D'$

Tenemos estudiando los casos del borde

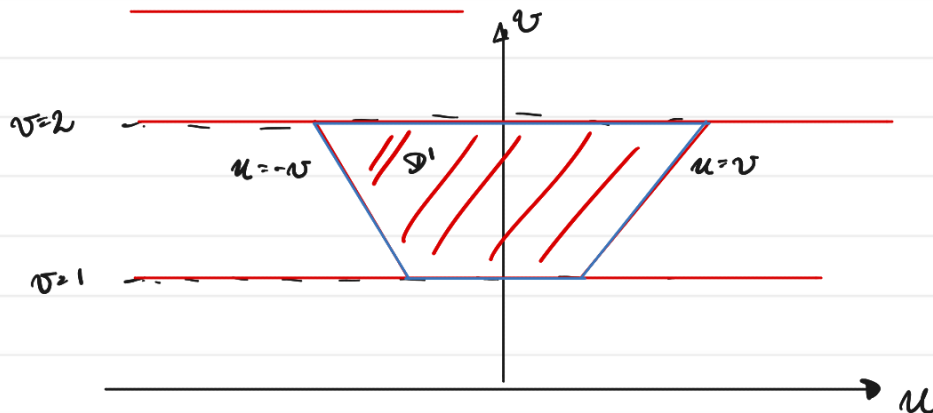
$$\left. \begin{array}{l} u = y - x \\ v = y + x \end{array} \right\} \begin{array}{l} x+y=1 \\ \xrightarrow{\quad} \\ v=1 \end{array} \quad \left. \begin{array}{l} u = y - x \\ v = y + x \end{array} \right\} \begin{array}{l} x+y=2 \\ \xrightarrow{\quad} \\ v=2 \end{array}$$

luego $1 \leq v \leq 2$

$$\left. \begin{array}{l} u = y - x \\ v = y + x \end{array} \right\} \begin{array}{l} x=0 \\ \xrightarrow{\quad} \\ u=y \end{array} \rightarrow \underline{u=v}$$

$$\left. \begin{array}{l} u = y - x \\ v = y + x \end{array} \right\} \begin{array}{l} y=0 \\ \xrightarrow{\quad} \\ u=-x \end{array} \rightarrow \underline{u=-v}$$

Tenemos $-v \leq u \leq v$



luego, por FCU,

$$\begin{aligned} \iint_D \cos\left(\frac{y-x}{y+x}\right) dA &= \int_1^2 \int_{-v}^v \cos\left(\frac{u}{v}\right) \cdot \frac{1}{2} du dv \\ &= \frac{1}{2} \int_1^2 v \left[\sin\left(\frac{u}{v}\right) \right] \Big|_{-v}^v = \frac{1}{2} \int_1^2 v (\sin(1) - \sin(-1)) dv \\ &= \sin(1) \int_1^2 v dv = \sin(1) \frac{v^2}{2} \Big|_1^2 = \frac{3}{2} \sin(1) \end{aligned}$$