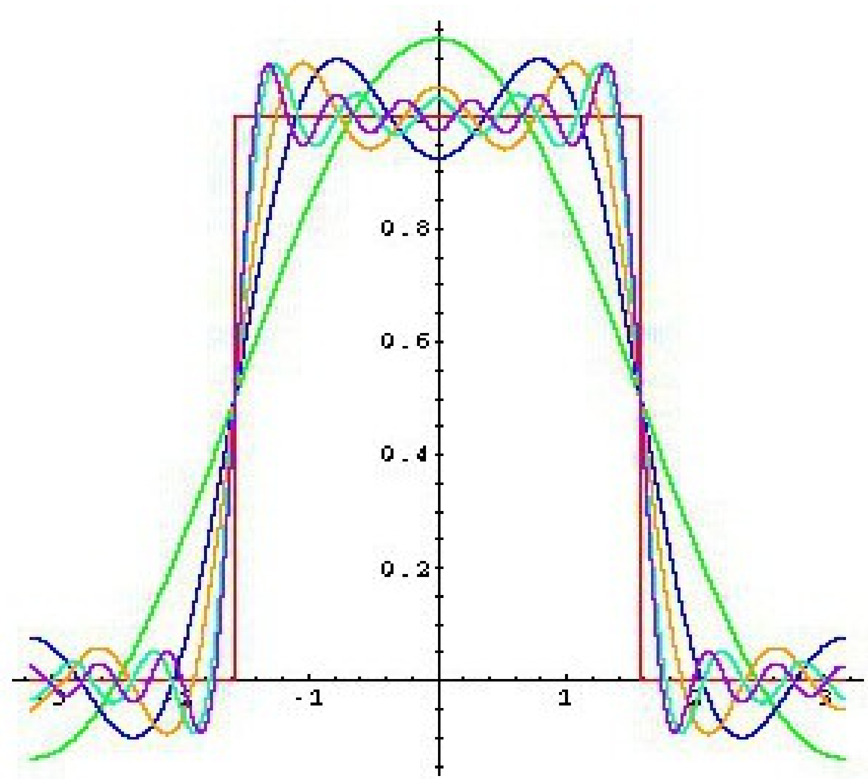
Serie de Fourier

A finales del siglo XVIII Jan Baptiste Joseph Fourier (1768-1830) descubrió un método que permite aproximar funciones periódicas mediante combinación lineal de funciones trigonométricas sencillas.



Serie de Fourier

Definición: Se llama serie de Fourier de una función f(x) en el intervalo [−L, L] a:

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \operatorname{sen} \frac{n\pi}{L} x \right)$$

Donde los coeficientes $\mathbf{a_0}$, $\mathbf{a_n}$ y $\mathbf{b_n}$ deben ser determinados.

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Los coeficientes $\mathbf{a_0}$, $\mathbf{a_n}$ y $\mathbf{b_n}$ están dados por:

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x) \, \mathrm{d}x$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi}{L} x \, \mathrm{d}x$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \operatorname{sen} \frac{n\pi}{L} x \, \mathrm{d}x$$

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$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi}{L} x \, \mathrm{d}x \qquad a_0 = \frac{1}{L} \int_{-L}^{L} f(x) \, \mathrm{d}x$$

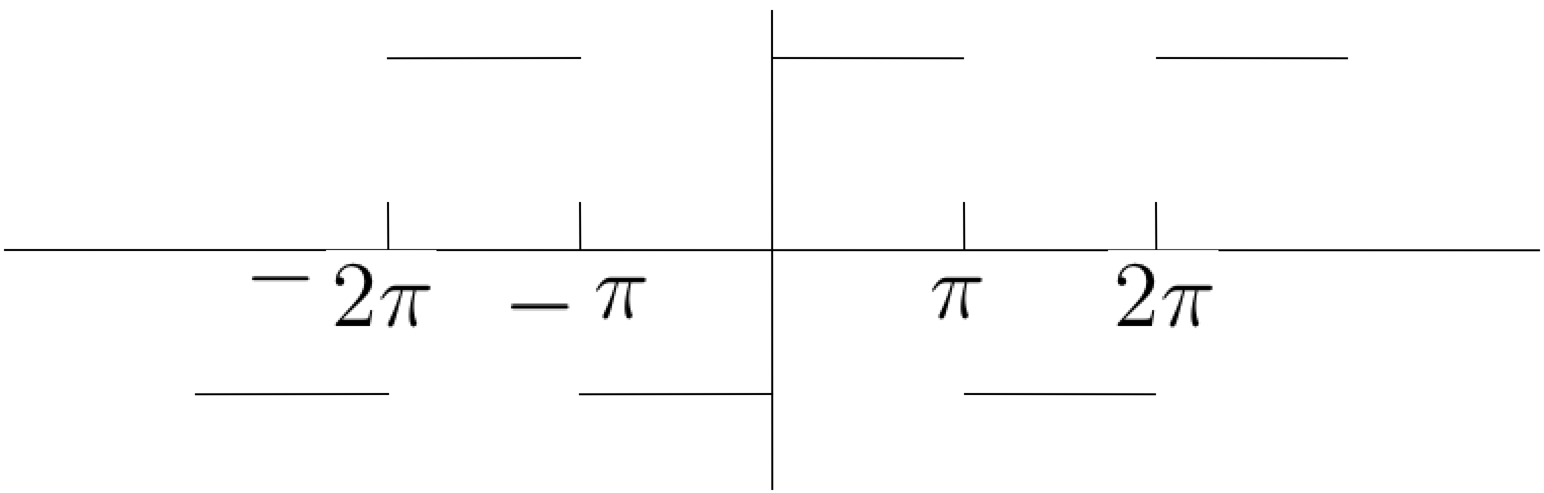
$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \operatorname{sen} \frac{n\pi}{L} x \, \mathrm{d}x$$

$$\cos n\pi = (-1)^n \qquad \operatorname{sen} n\pi = 0$$

Serie de Fourier

Ejemplo: consideremos la función:

$$f(x) = \begin{cases} 1, & \text{si} \quad 0 \le x \le \pi; \\ -1, & \text{si} \quad \pi < x < 2\pi, \end{cases}$$



En este caso $2L = 2\pi$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cos(nx)dx$$

y considerando que $f(x) = -1$ entre $-\pi$ y π

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cos(nx)dx$$

$$a_n = \frac{1}{\pi}[-\int_{-\pi}^{0} f(x)cos(nx)dx + \int_{0}^{\pi} f(x)cos(nx)dx]$$

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$$\int cos(nx)dx = \frac{1}{n}sen(nx)$$

evaluada en $0, \pi$ ó $-\pi$ es igual a 0, por lo tanto:

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) sen(nx) dx$$

$$b_n = \frac{1}{\pi}[-\int_{-\pi}^{0} sen(nx)dx + \int_{0}^{\pi} sen(nx)dx]$$

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$$-\int_{-\pi}^{0} sen(nx)dx = \frac{1}{n}cos(nx)|_{-\pi}^{0} = \frac{1}{n} - \frac{1}{n}cos(-n\pi)$$

$$\int_{0}^{\pi} sen(nx)dx = \frac{-1}{n}cos(nx)|_{0}^{\pi} = \frac{-1}{n}cos(n\pi) - \frac{-1}{n}$$

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$$b_n = \frac{1}{\pi}[\frac{1}{n} - \frac{1}{n}cos(-n\pi) + \frac{1}{n} - \frac{1}{n}cos(n\pi)]$$

$$b_n = \frac{1}{\pi}[\frac{2}{n} - \frac{2}{n}cos(n\pi)]$$

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$$b_n = \frac{1}{\pi}[\frac{2}{n} - \frac{2}{n}cos(n\pi)]$$

$$b_n = \frac{2}{n\pi}[1 - cos(n\pi)]$$

$$cos(n\pi) = +1, \quad n \quad par$$

$$cos(n\pi) = -1, \quad n \quad impar$$

$$b_n = \frac{1}{\pi}[-\int_{-\pi}^{0} sen(nx)dx + \int_{0}^{\pi} sen(nx)dx]$$

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$$cos(n\pi) = +1, \qquad n \qquad par$$

$$cos(n\pi) = -1, \qquad n \qquad impar$$

$$b_n = \frac{4}{n\pi}, \qquad n \qquad impar$$

$$a_0 = 0 \quad a_n = 0 \quad b_n = \frac{4}{n\pi}, \quad n \quad impar \quad 2L = 2\pi$$

$$f(x) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \operatorname{sen} \frac{n\pi}{L} x \right)$$

$$f(x) = \frac{4}{\pi} sen(x) + \frac{4}{3\pi} sen(3x) + \frac{4}{5\pi} sen(5x) + \cdots$$

$$f(x) = \frac{4}{\pi}[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots]$$

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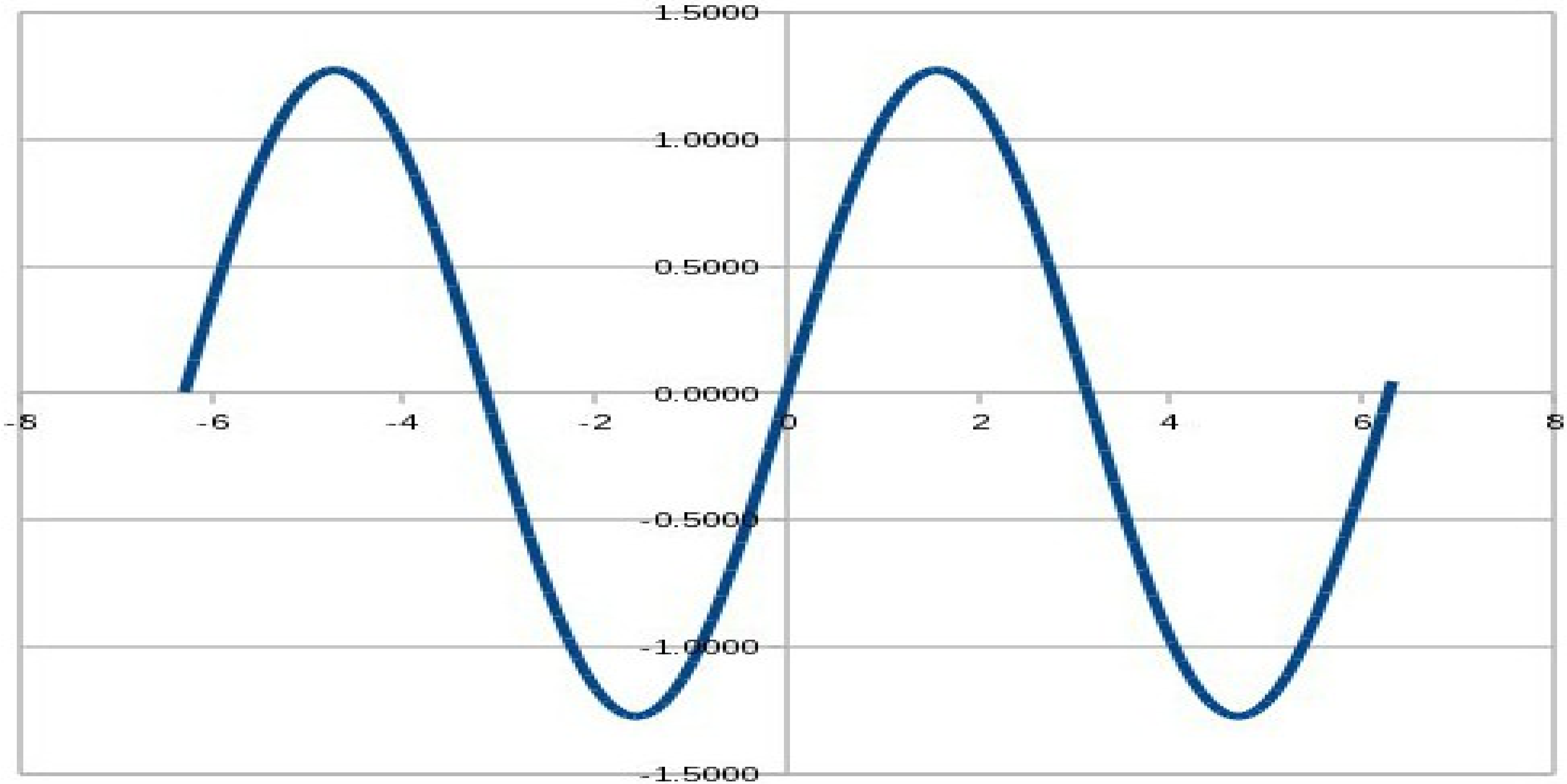
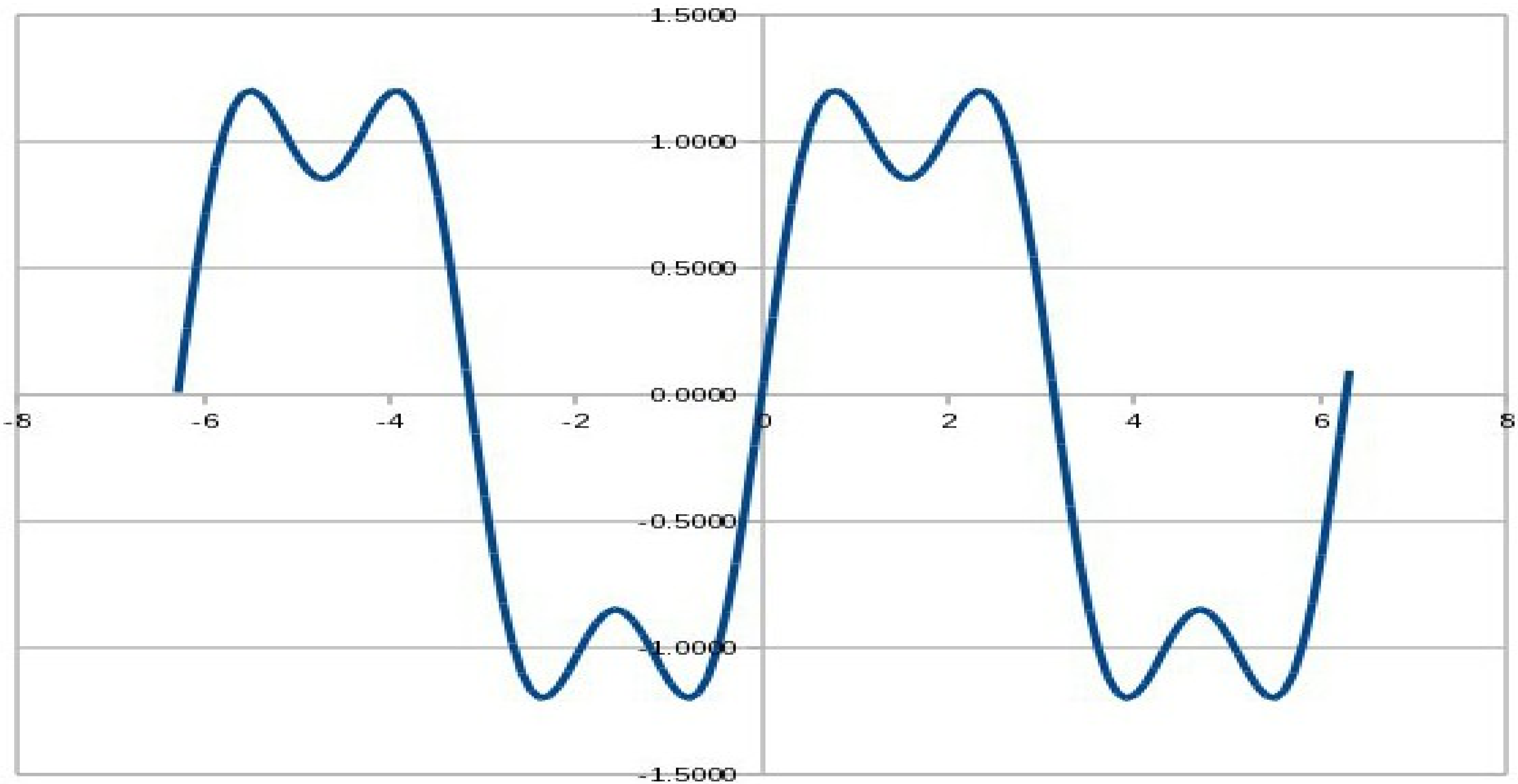
Sumando 1 término de la serie:



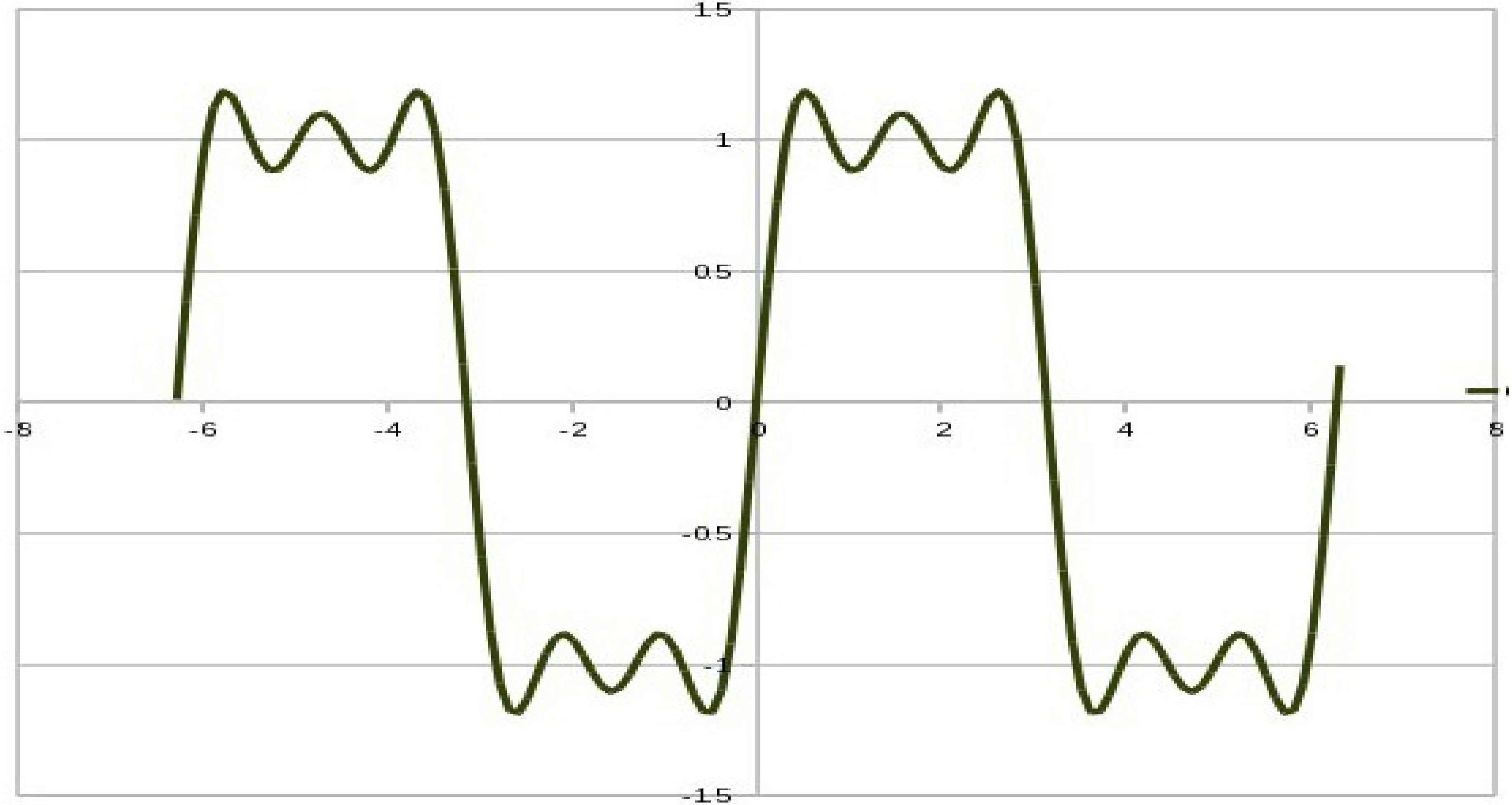
$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$

Sumando 2 términos de la serie:



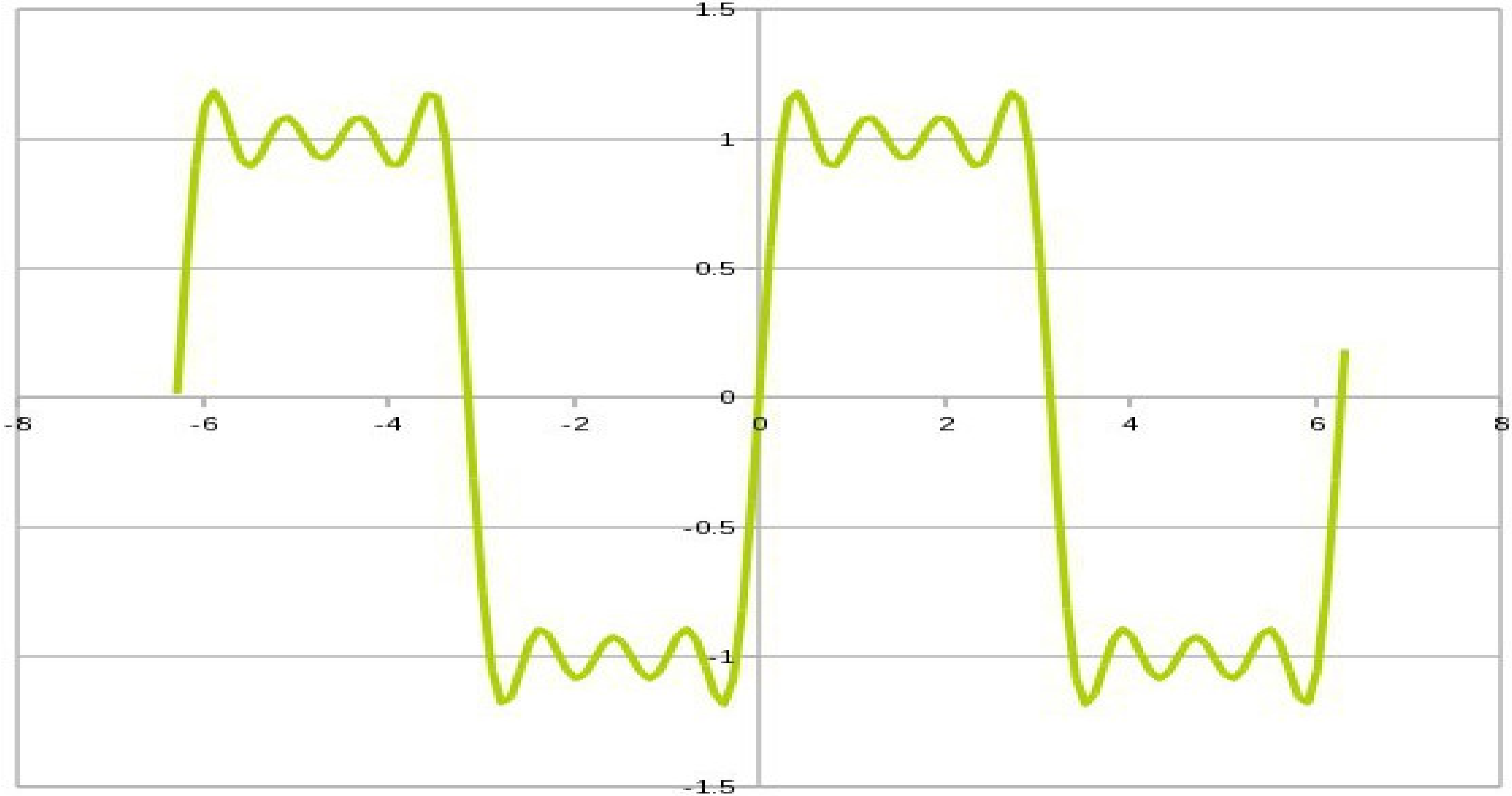
$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$

Sumando 3 términos de la serie:

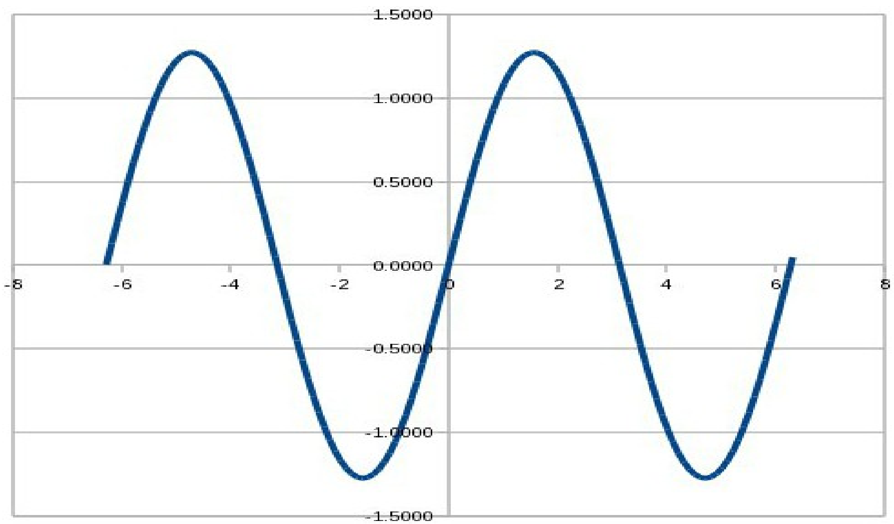


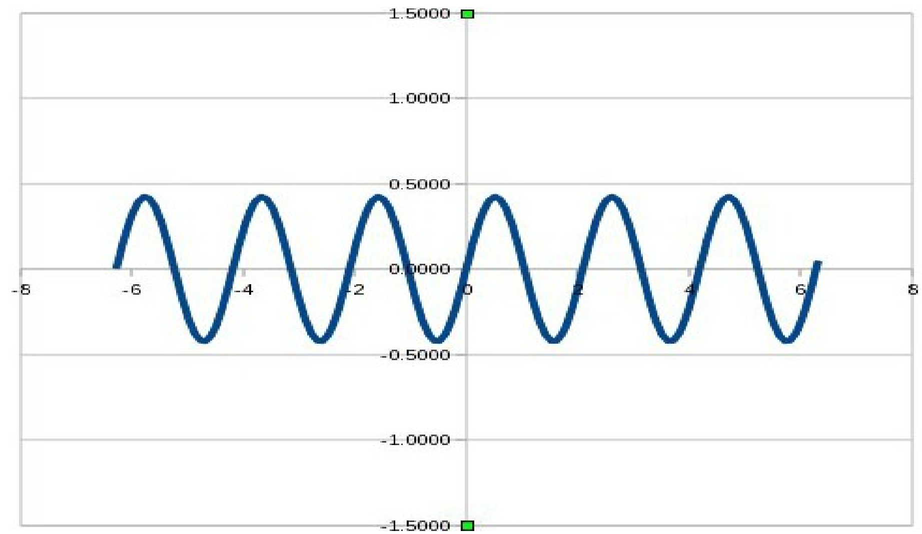
$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$

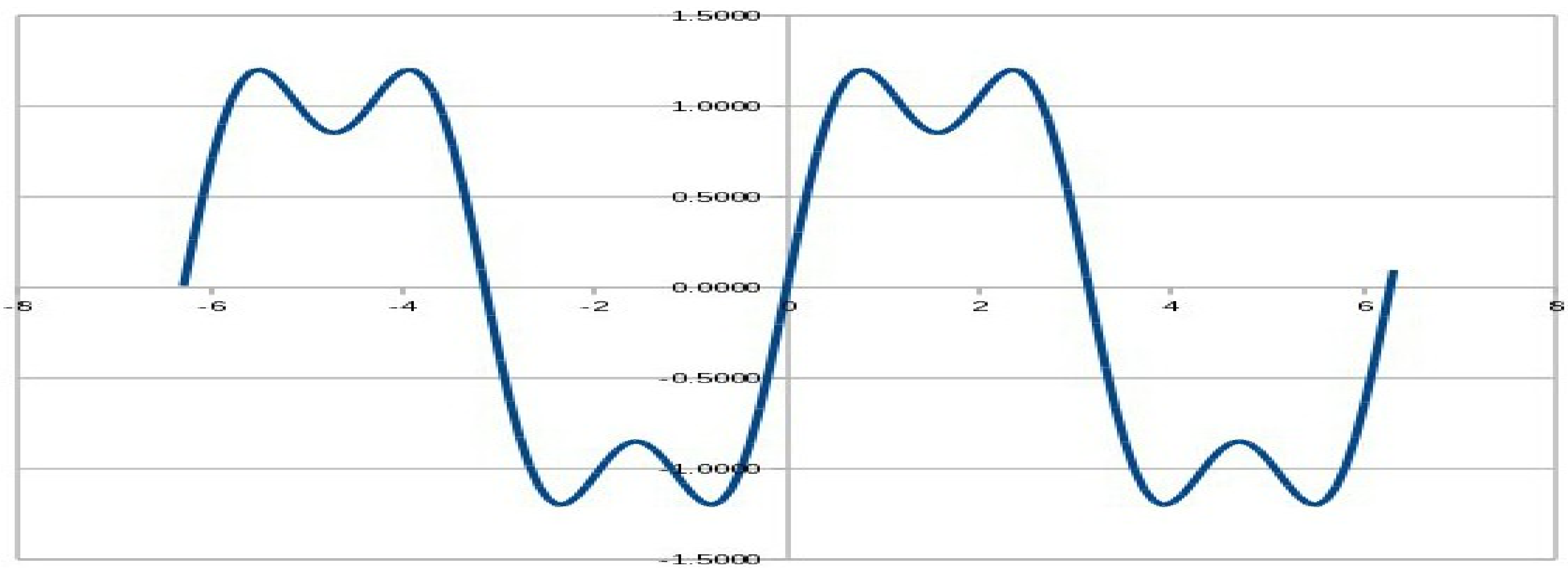
Sumando 4 términos de la serie:



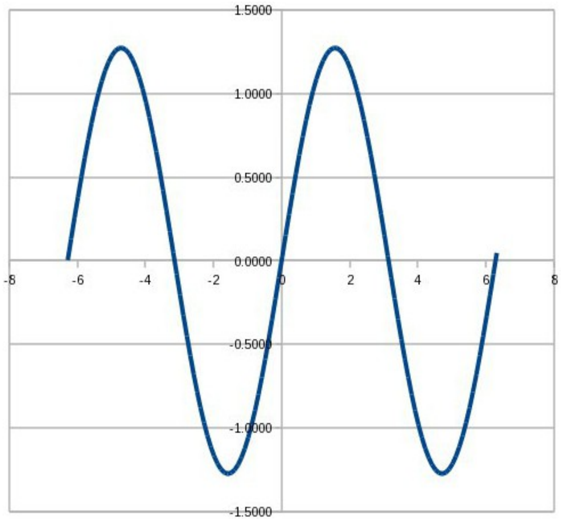
$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$

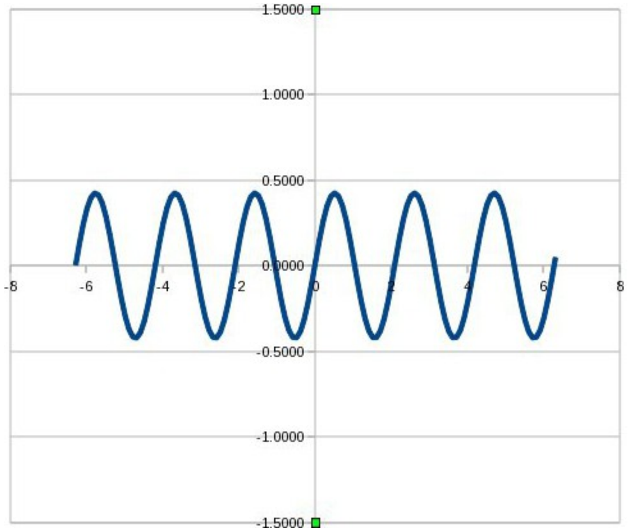


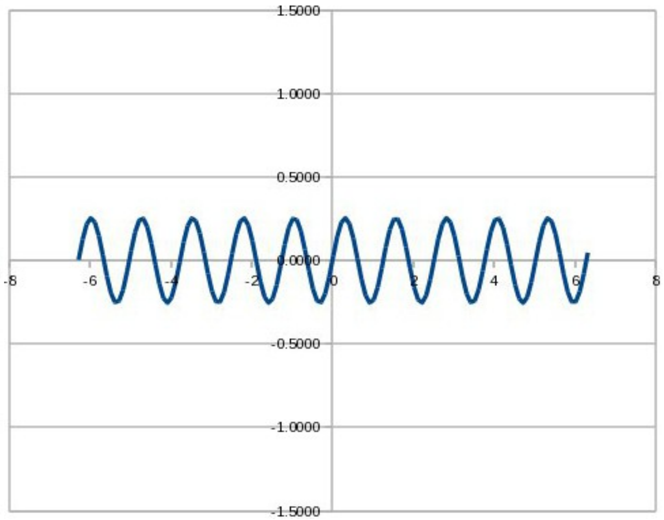


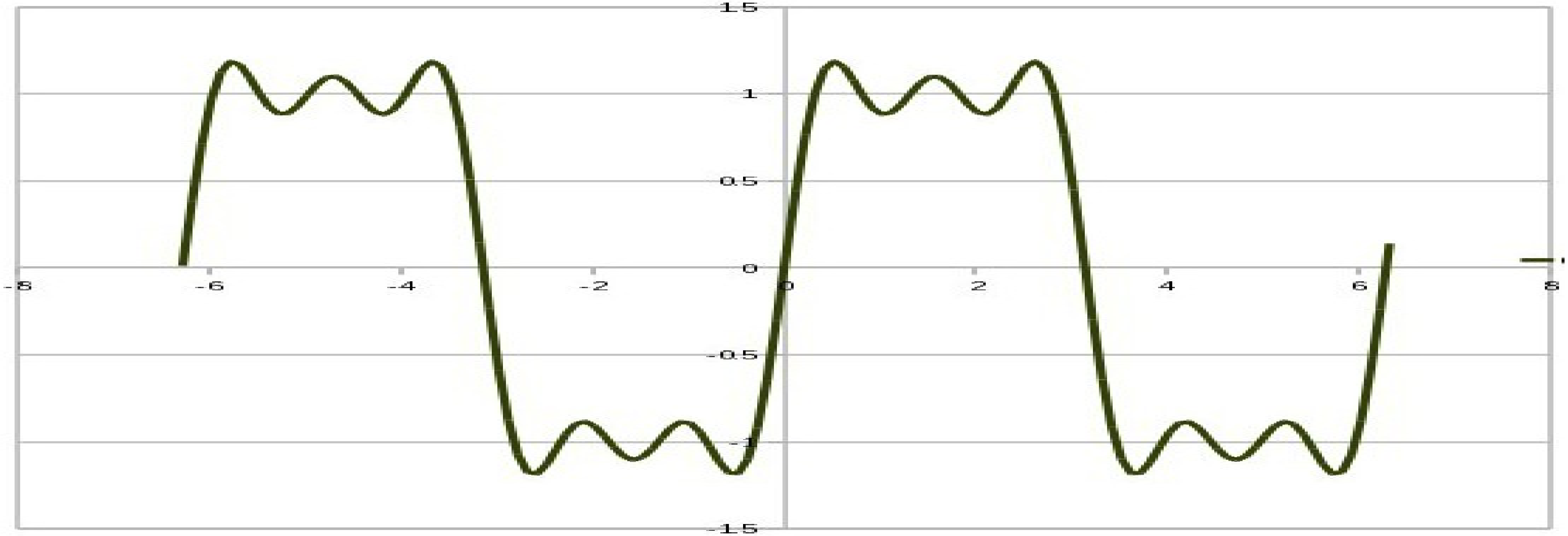


$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$









$$f(x) = \frac{4}{\pi}\left[sen(x) + \frac{sen(3x)}{3} + \frac{sen(5x)}{5} + \cdots\right]$$